

# Solving Systems of Linear Equations Substitutions

## Student/Class Goal

Students thinking about continuing their academic studies in a post-secondary institution will need to know and be able to do problems on solving systems of equations.

## Outcome *(lesson objective)*

Students will accurately solve a system of equations algebraically using substitution.

## Time Frame

1-2 hours

## Standard *Use Math to Solve Problems and Communicate*

## NRS EFL 5-6

Number Sense	Benchmarks	Geometry & Measurement	Benchmarks	Processes	Benchmarks
Words to numbers connection		Geometric figures		Word problems	
Calculation		Coordinate system		Problem solving strategies	
Order of operations		Perimeter/area/volume formulas		Solutions analysis	
Compare/order numbers		Graphing two-dimensional figures		Calculator	5.28, 5.29
Estimation		Measurement relationships		Math terminology/symbols	
Exponents/radical expressions		Pythagorean theorem		Logical progression	
<b>Algebra &amp; Patterns</b>	<b>Benchmarks</b>	Measurement applications		Contextual situations	
Patterns/sequences		Measurement conversions		Mathematical material	
Equations/expressions	5.16, 6.16	Rounding		Logical terms	
Linear/nonlinear representations		<b>Data Analysis &amp; Probability</b>	<b>Benchmarks</b>	Accuracy/precision	
Graphing		Data interpretation		Real-life applications	
Linear equations	5.19, 6.19	Data displays construction		Independence/range/fluency	
Quadratic equations		Central tendency			
		Probabilities			
		Contextual probability			

## Materials

Paper and pencils  
*Steps to Solve a System of Equations by Substitution* Handout  
*Using Substitution to Solve Systems of Equations* Activity  
*Math Scavenger Hunt* Teacher Resource  
*Math Scavenger Hunt* Student Answer Sheet  
*Substitution Scavenger Hunt* Information Sheet/Answers

## Learner Prior Knowledge

Students should be fluent in solving linear equations and integer operations. They should have the ability to solve systems of equations by graphing. They will have completed earlier lessons on systems of equations, such as *Solving Systems of Linear Equations Graphing*.

**Teacher Note** Be sure to classify each system as consistent or inconsistent and dependent or independent.

## Instructional Activities

Step 1 - On the board, write the sample system ( $-x + 2y = 4$  and  $5x - 3y = 1$ ) found on handout, *Steps to Solve a System of Equations by Substitution*. Ask the class how they might solve the system of equations (graphing). Remind the class that this is the method that they just studied in class.

Ask the class if they know of any other methods to solve this system /problem. Ask if they think the system could be solved algebraically. Using the steps outlined on the handout for solving a system of equations by substitution, demonstrate how to solve this system of equations by substitution.

Step 2 - Distribute copies of *Steps to Solve a System of Equations by Substitution* handout. Ask the students to review the steps on the handout. Using the three sample systems on the handout, teacher works through each example with student input, asking questions:

- Which equation would be best to rewrite for substitution?
- Which variable do we want to solve for first?
- What does the equation look like after we substitute?
- Do the solutions work for both equations?

Step 3 - Working in pairs, students will complete the *Using Substitution to Solve Systems of Equations* activity. Students will match the systems of equations with the correct equation modified for substitution.

Step 4 - After verifying that the students have selected the correct modified equations on the worksheet, solve the systems of equations found on *Using Substitution to Solve Systems of Equations*.

Step 5 - Set up a Math Scavenger Hunt in the classroom using the directions found in the *Math Scavenger Hunt* Teacher Resource and Answer Sheet. The questions and answers found in the *Substitution Scavenger Hunt* Information Sheet will provide the questions and solutions to place on each clue card. The answers are also included for your reference.

**Assessment/Evidence** *(based on outcome)*

Math Scavenger Hunt completed with 100% accuracy.

**Teacher Reflection/Lesson Evaluation**

*This lesson has not yet been field tested.*

**Next Steps**

This is part of a series of lessons on solving systems of linear equations. To continue the study, complete *Solving Systems of Linear Equations Elimination*.

**Technology Integration**

Systems of Linear Equations: Solving by Substitution <http://www.purplemath.com/modules/systlin4.htm>

Solving Systems of Equations by Substitution <http://cstl.syr.edu/fipse/algebra/unit5/subst.htm>

**Purposeful/Transparent**

Transitions classes focus on higher level math skills, such as solving systems of equations that will be required for college classes.

**Contextual**

Students will determine the best method to use when solving systems of equations as they solve problems using graphing, substitution, and elimination/addition.

**Building Expertise**

Students are given ample practice working in graphing, substitution, and elimination/addition of linear equations and write a system of equations when solving a mathematical situation.

## Steps to Solve a System of Equations by Substitution

1. Solve one equation for one of the variables.
2. Substitute the resulting expression in the other equation.
3. Solve the resulting equation for the first variable.
4. Find the values of the variables by substituting the solution back into the original equation to solve for the second variable.
5. Check the solution in both equations of the system.

### Example

$$\begin{aligned} -x + 2y &= 4 \\ 5x - 3y &= 1 \end{aligned}$$

#### Step 1

*Since the first equation has a term with a coefficient of -1 or 1, solve the first equation for x*

$$\begin{aligned} -x + 2y - 2y &= 4 - 2y && \text{(subtract } 2y \text{ from both sides)} \\ -x &= 4 - 2y && \text{(result after subtracting } 2y \text{ from both sides)} \\ -x/-1 &= (4 - 2y)/-1 && \text{(divide both sides by } -1 \text{ so } x \text{ is positive)} \\ X &= 2y - 4 && \text{(flipped the order of the terms)} \end{aligned}$$

#### Step 2 (substitute for x)

$$5(2y - 4) - 3y = 1 \quad \text{(Substitute the resulting expression into the other equation)}$$

#### Step 3 (solve for y)

$$\begin{aligned} 10y - 20 - 3y &= 1 && \text{(distribute the 5)} \\ 7y - 20 &= 1 && \text{(combine like terms)} \\ 7y - 20 + 20 &= 1 + 20 && \text{(add 20 to both sides)} \\ 7y &= 21 \\ 7y/7 &= 21/7 && \text{(divide both sides by 7)} \\ y &= 3 \end{aligned}$$

#### Step 4 (solve for x)

$$\begin{aligned} -x + 2y &= 4 && \text{(original equation)} \\ -x + 2(3) &= 4 && \text{(y value substituted into the equation)} \\ -x + 6 &= 4 \\ -x + 6 - 6 &= 4 - 6 && \text{(subtract 6 from both sides)} \\ -x &= -2 && \text{(combine like terms)} \\ -x/-1 &= -2/-1 && \text{(divide both sides by } -1) \\ X &= 2 \end{aligned}$$

### Step 5 (check your answer)

$$\begin{aligned} -x + 2y &= 4 \\ -(2) + 2(3) &= 4 \\ -2 + 6 &= 4 \\ 4 &= 4 \end{aligned}$$

$$\begin{aligned} 5x - 3y &= 1 \\ 5(2) - 3(3) &= 1 \\ 10 - 9 &= 1 \\ 1 &= 1 \end{aligned}$$

*(Results of substituting the solutions into both equations)*

### Sample Systems to Solve

$$\begin{aligned} 5x - y &= 1 \\ 3x + 2y &= 13 \end{aligned}$$

$$\begin{aligned} r &= 5 - s \\ 2r + 7s &= 0 \end{aligned}$$

$$\begin{aligned} 4a + b - 8 &= 0 \\ 5a + 3b - 3 &= 0 \end{aligned}$$

## Using Substitution to Solve Systems of Equations Activity

**Directions** Match the system of equations with the modified equation that can be used to solve the system of equations by substitution. Draw a line between the system and the equation used to substitute.

$$\begin{aligned}2x + y &= 11 \\ x - y &= 2\end{aligned}$$

$$x = -2y + 6$$

$$\begin{aligned}4x - y &= 7 \\ 5x - 8y &= 2\end{aligned}$$

$$x = -6y + 5$$

$$\begin{aligned}2x + 2y &= 4 \\ 3x - 3y &= 18\end{aligned}$$

$$y = -2x + 1$$

$$\begin{aligned}2x + y &= 1 \\ 10x - 4y &= 2\end{aligned}$$

$$y = 4x - 7$$

$$\begin{aligned}-3x - y &= -13 \\ X + 2y &= 6\end{aligned}$$

$$x = y + 2$$

$$\begin{aligned}2x - 6y &= 4 \\ X + 6y &= 5\end{aligned}$$

$$x = -y + 2$$

## Activity Answers

$$\begin{aligned}2x + y &= 11 \\ x - y &= 2\end{aligned}$$

$$x = y + 2$$

$$\begin{aligned}4x - y &= 7 \\ 5x - 8y &= 2\end{aligned}$$

$$y = 4x - 7$$

$$\begin{aligned}2x + 2y &= 4 \\ 3x - 3y &= 18\end{aligned}$$

$$x = -y + 2$$

$$\begin{aligned}2x + y &= 1 \\ 10x - 4y &= 2\end{aligned}$$

$$y = -2x + 1$$

$$\begin{aligned}-3x - y &= -13 \\ X + 2y &= 6\end{aligned}$$

$$x = -2y + 6$$

$$\begin{aligned}2x - 6y &= 4 \\ X + 6y &= 5\end{aligned}$$

$$x = -6y + 5$$

# Math Scavenger Hunt Teacher Resource

## Supplies

Paper or card stock (8 ½" by 11")

2 colored markers

Tape

A math scavenger hunt is a fun way to assess the math skills of your students. Most any math topic can be evaluated with this activity, and the students will stay active as they move around the room solving problems and searching for the answers. Students can work in groups or alone as they complete the activity.

To set up a scavenger hunt select 6-8 problems with answers. Before you make the scavenger hunt clue cards, do some planning to make sure each problem and its answer will be on different cards. This has already been done for you in the series of lessons on systems of equations. When you have decided on the problem and answer to place on each card, write a problem at the top (portrait orientation) of the clue card and a solution at the bottom of the card. Write all the answers in one color of marker, and use the second color for the problems. Tape these sheets around the room.

## Math Clue Card Example

$2 \times 4$
10

Now it is time for the students to complete the Math Scavenger Hunt. Give each student a Scavenger Hunt Answer Sheet (see below). Students can start their hunt at any location in the room. This way the class will be spread out around the classroom. At their first stop, the students will write the problem on their answer sheet and solve it. Remember the problem will be at the bottom of the sheet. There is space on the answer sheet for the students to show their work. Once they have solved this problem they will find the Scavenger Hunt Clue Card with their answer. The problem at the bottom of this clue card will be the students' next problem to solve. If the students don't find their answer when they look around the room, the students know to redo their work. Students continue with this process until all the problems have been completed, and they return to the card which contains their first problem.

The answers can be corrected quickly because the answers will be in a specific order. Remember each student will start the Scavenger Hunt in a different place in the answer sequence.

# Math Scavenger Hunt Student Answer Sheet

Problems	Answers



## Substitution Scavenger Hunt Information Sheet

Use the following systems of equations and solutions to create a Math Scavenger Hunt for the students. The systems in the left column should be placed on the same card as the solutions next to them. **Note** The solutions do not match the systems they are next to!

$$\begin{array}{l} X - 2y = 0 \\ 2x - 5y = -4 \end{array} \quad (12, 17)$$

$$\begin{array}{l} -1/2 x - y = -3 \\ X + 3y = 6 \end{array} \quad (4, 1)$$

$$\begin{array}{l} y = 8 - x \\ 4x - 3y = -3 \end{array} \quad (2, 3)$$

$$\begin{array}{l} x = 8y \\ x - 4y = 12 \end{array} \quad (6, 0)$$

$$\begin{array}{l} y = x + 5 \\ y = 2x - 7 \end{array} \quad (24, 3)$$

$$\begin{array}{l} -x + 2y = 4 \\ 5x - 3y = 1 \end{array} \quad (2, 1)$$

$$\begin{array}{l} -3x - y = -13 \\ x + 2y = 6 \end{array} \quad (3, 5)$$

$$\begin{array}{l} 4x - y = 7 \\ 5x - 8y = 2 \end{array} \quad (4, 4)$$

## Substitution Scavenger Hunt Information Sheet Answers

Use the following systems of equations and solutions to create a Math Scavenger Hunt for the students. The systems in the left column should be placed on the same card as the solutions next to them.

$$\begin{aligned}x - 2y &= 0 \\ 2x - 5y &= -4\end{aligned}$$

$$\begin{aligned}-\frac{1}{2}x - y &= -3 \\ x + 3y &= 6\end{aligned}$$

$$\begin{aligned}y &= 8 - x \\ 4x - 3y &= -3\end{aligned}$$

$$\begin{aligned}x &= 8y \\ x - 4y &= 12\end{aligned}$$

$$\begin{aligned}y &= x + 5 \\ y &= 2x - 7\end{aligned}$$

$$\begin{aligned}-x + 2y &= 4 \\ 5x - 3y &= 1\end{aligned}$$

$$\begin{aligned}-3x - y &= -13 \\ x + 2y &= 6\end{aligned}$$

$$\begin{aligned}4x - y &= 7 \\ 5x - 8y &= 2\end{aligned}$$

$$(12, 17)$$

$$(4, 1)$$

$$(2, 3)$$

$$(6, 0)$$

$$(24, 3)$$

$$(2, 1)$$

$$(3, 5)$$

$$(4, 4)$$