Solving Systems of Equations
Introduction

Student/Class Goal
Students thinking about continuing their academic studies in a post-secondary institution will need to know and be able to do problems on solving systems of equations.

Outcome (*lesson objective)*
Students will write simple systems of equations and become familiar with systems of equations vocabulary terms.

Time Frame
1 hour

Standard *Use Math to Solve Problems and Communicate*

NRS EFL 5-6

Number Sense	Benchmarks	Geometry & Measurement	Benchmarks	Processes	Benchmarks
Words to numbers connection		Geometric figures		Word problems	
Calculation		Coordinate system		Problem solving strategies	
Order of operations		Perimeter/area/volume formulas		Solutions analysis	5.27, 6.28
Compare/order numbers		Graphing two-dimensional figures		Calculator	
Estimation		Measurement relationships		Math terminology/symbols	5.29, 6.30
Exponents/radical expressions		Pythagorean theorem		Logical progression	
Algebra & Patterns	**Benchmarks**	Measurement applications		Contextual situations	
Patterns/sequences		Measurement conversions		Mathematical material	
Equations/expressions	5.16, 6.16	Rounding		Logical terms	
Linear/nonlinear representations	5.17, 6.17	**Data Analysis & Probability**	**Benchmarks**	Accuracy/precision	
Graphing		Data interpretation		Real-life applications	
Linear equations	5.19, 6.19	Data displays construction		Independence/range/fluency	
Quadratic equations		Central tendency			
		Probabilities			
		Contextual probability			

Materials
8 ½" by 11" card stock or paper
2 colored markers
Writing & Solving Algebraic Equations & Expressions Handout
Writing Systems of Equations Handout
Systems of Equations Flowchart
Vocabulary Sheet Teacher Resource

Learner Prior Knowledge
Students will understand and be able to solve linear equations and turn a verbal story into an algebraic sentence.

Instructional Activities
Step 1 - Read and discuss the problem situations found on the *Writing & Solving Algebraic Equations & Expressions* handout. As a class or in small groups write an equation for each problem situation and solve it. Be sure to discuss what was the unknown quantity they were solving for in each problem. Discuss that in each equation there was one unknown they were looking for.

Read the following situation to the students: *The cost to attend a football game is $6.00 for adults and $2.00 for students. There were 364 tickets sold at the game. If the sales for the tickets totaled $1348, what was the number of adult tickets sold and the number of student tickets sold?*

Ask the students what are we trying to find out in this problem? (number of adult and student tickets) Be sure the students recognize that unlike the previous problems, there are two unknowns to solve for in this problem. Next ask the students to write equations based on the information in the problem. You might name the variables "a" and "s" for adult tickets and student tickets. Equations for this problem would be: *a + s = 364 total tickets* and *$6a + $2s = $1348.*

Step 2 - Read 2 more problem solving situations to the class where they need to write the equations to go with the "story problem"

situation. Two possible examples follow:

Amish Country Inn offers vacation packages during the winter. The "Mini-retreat" package includes a 4-night stay and 8 meals for $360. The "Weekender" package includes a 2-night stay and 6 meals for $220. What is the room rate at the Amish Country Inn for one night? What is the cost of a meal at the inn?

The nursing home has a total of 22 nurses and nurses' aides working each shift. There are 6 more aides than there are nurses. How many nurses and nurses' aides work during each shift?

Discuss the students' equations and expressions from the three problems above. We have just written a **system of equations** for each of the previous 3 problems where there are two unknowns.

Step 3 – Complete the exercises in the handout, *Writing Systems of Equations*. These problems should not be solved! Make sure students are identifying the two variables and writing the equations correctly. Monitor the class to make sure the students are identifying the systems correctly.

Step 4 – Before we learn ways to solve systems of equations, it is important to study the types of solutions that are possible. There are three possible **solutions** to a system of equations: one solution, many/all solutions, no solutions. Mathematicians classify these solutions into different categories. A system is **consistent** if there is at least one solution. A system is **inconsistent** if there are no solutions. Consistent systems can be dependent or independent. **Dependent** systems have all solutions in common. When graphed, the two lines are the same. **Independent** systems have only one solution in common. This means when graphed, the two lines cross at one point.

Step 5 - Distribute and complete the *Systems of Equations* Flowchart in pairs. Also give student the *Vocabulary Sheet* Teacher Resource with definitions of systems of linear equations to use as a resource. Using a white board or overhead projector, create the flowchart diagram, having students supply the correct terms from the word bank.

Step 6 - Mention the three strategies to solve systems of equations they will be learning in upcoming lessons [graphing, substitution and elimination/addition].

Assessment/Evidence (*based on outcome)*
Writing & Solving Algebraic Equations & Expressions Handout
Writing Systems of Equations Handout
Systems of Equations Flowchart

Teacher Reflection/Lesson Evaluation
This lesson has not yet been field-tested.

Next Steps
This is part of a series of lessons on solving systems of linear equations. To continue the study, complete *Solving Systems of Linear Equations Graphing*.

Technology Integration
Systems of Linear Equations: Definitions http://www.purplemath.com/modules/systlin1.htm
Solving Systems of Equations (Simultaneous Equations)
http://www.cliffsnotes.com/study_guide/Solving-Systems-of-Equations-Simultaneous-Equations-.topicArticleId-9046,articleId-9042.html
Solving Systems of Multivariate Equations http://cstl.syr.edu/fipse/algebra/unit5/multisys.htm

Purposeful/Transparent
Transitions classes focus on higher level math skills, such as solving systems of equations that will be required for college classes.

Contextual
Students will determine the best method to use when solving systems of equations as they solve problems using graphing, substitution, and elimination/addition.

Building Expertise
Students are given amble practice working in graphing, substitution, and elimination/addition of linear equations and write a system of equations when solving a mathematical situation.

Writing & Solving Algebraic Equations & Expressions

Directions Identify the variable and write an algebraic equation or expression for each situation. Solve your equation.

1. A shipping clerk must send packages of chemicals to a laboratory. The container she is using to ship the chemicals will hold 18 lbs. Each package of chemicals weighs 3 lbs. How many packages of chemicals will the container hold?

2. The shipping clerk in problem 1 (above) must send 220 packages of chemicals to a laboratory in Wooster and 55 packages to a laboratory in Columbus. How many containers will she need to send the chemicals to the two locations?

3. When a fair skinned person visits "I Want a Tan Tanning Salon" they are advised to start with 10 minutes of exposure and gradually build up to a maximum of 22 minutes of exposure, increasing exposure 1 minute every 2 days. Write an equation to determine the minutes of exposure a fair person can experience after n days. How many minutes could be experienced after 1 week? Two weeks? A month?

4. Latasha wants to buy amaryllis bulbs to give to all her friends. She plans to purchase the bulbs from a distributor in Holland, Michigan. The bulbs cost $5 each and shipping is 20% of the total order. How much Latasha will pay for b bulbs?

Writing Systems of Equations

Directions Identify two variables in each problem. Write the equations suggested/required by the "story" situation. DO NOT SOLVE THESE EQUATIONS!

1. The Browns scored 13 more points than the Saints. The total of their scores was 47. How many points did each team score?

2. A company produces telephones at the rate of 600 per day. A customer survey indicates that the demand for phones with built in answering machines is twice as great as the demand for phones without the machines. If you are deciding the production quota for the day, how many phones with answering machines would you schedule for production? How many without answering machines would you make?

3. Sarah is the director of the Hoonah marching band. She must order 35 new uniforms for the band. There are usually five more girls than twice the number of boys in the band. How many uniforms of each type should she order for the band?

4. Mary's children decide to run a lemonade stand to earn some extra money. The cost to start the business is $1.20 and each cup of lemonade costs 6 cents to make. If lemonade sells for 10 cents a cup, how many cups must Mary's children sell to make a profit?

5. At the "Great Hair Barber Shop" Nita and Joe do a total of 95 haircuts each week. If Nita does 16 fewer than twice as many as Joe, how many haircuts does each person do?

6. John has 6 puppies for sale and wants to advertise them in the Cleveland Plain Dealer. To advertise in the paper there is a flat or fixed rate for the first ten words of the ad and a fixed charge for each additional word. The cost of a 17-word ad is $14.55. The cost for a 21-word ad is $17.15. What is the flat rate for the first 10 words and the fixed charge for each additional word?

7. You are planning a huge graduation party for your son. You decide to offer both a beef and a chicken meal at the party. The chicken dish costs $5, and the beef dish cost $7. There will be 250 people at the party, and the total cost of the food is $1500. How many chicken meals will there be? How many beef meals will there be?

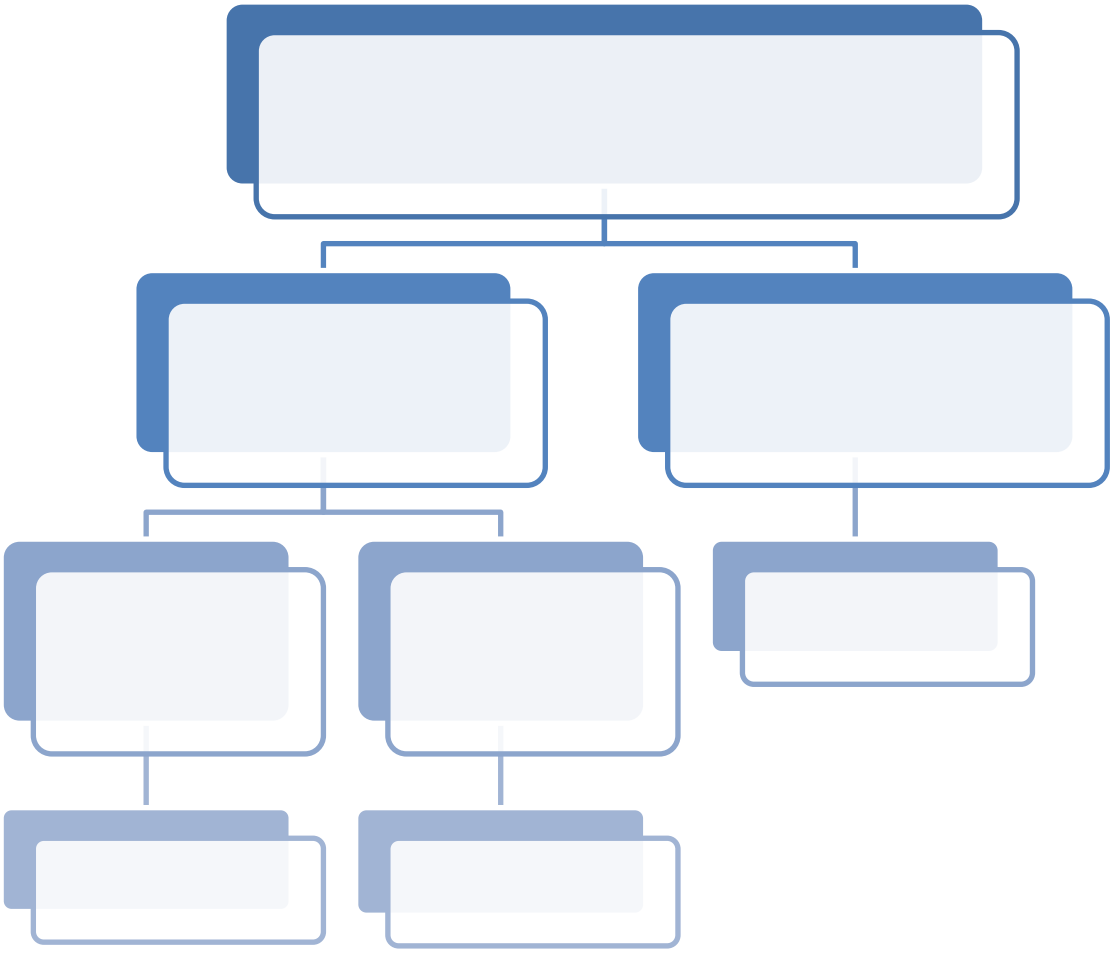
8. Paula needs to replace the floor in her family room since her cat peed in several places. She wants to put down both vinyl flooring and carpet in the room. The carpet she selected costs $2 per square foot. The vinyl floor covering costs $1 per square foot. She has $500 to spend on materials and must cover an area of 300 square feet. How much carpet and vinyl flooring will she buy to meet her requirements?

9. A salesperson at an electronics store is given a choice of two different compensation plans. Plan A pays him a weekly salary of $250 plus a commission of $25 for each stereo sold. Plan B offers no salary but pays $50 commission on each stereo sold. How many stereos must the salesperson sell to make the same amount of money with both plans? Write a paragraph answering the following questions: When is plan B the better plan? When is plan A the better plan? Which plan would you select and why?

10. ABLE Trucking Company has a job moving 21 tons of sand. The company has 8 drivers in the company and 2 types of trucks. One type of truck can haul 5 tons of sand and the other type of truck can haul 3 tons. Insurance requirements make it necessary for the trucks hauling 5 tons of gravel to have two drivers in the cab during operation. Three ton trucks require only one driver. Using all available drivers, how many trucks of each size will be needed to move the sand in one trip?

Systems of Equations Flowchart

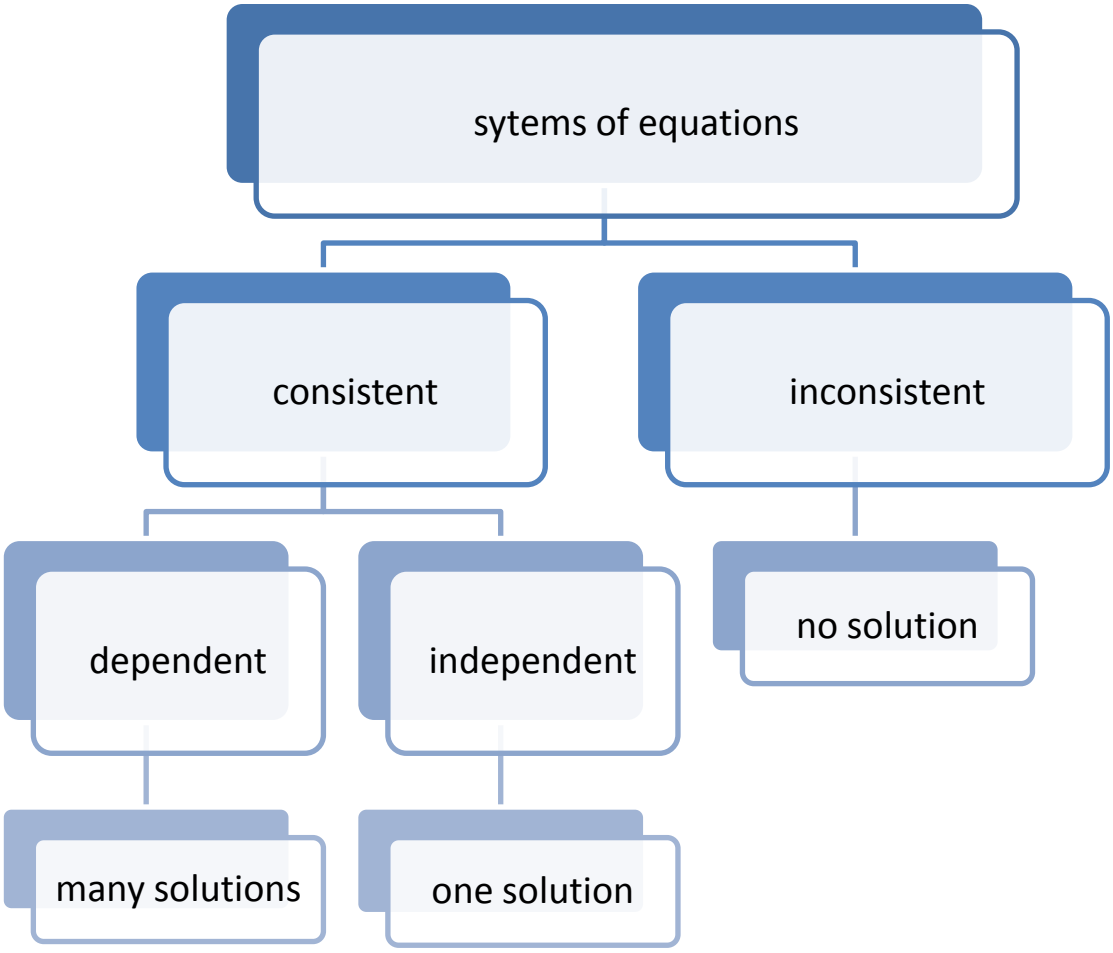
Directions Place the word (from below) in the box that best shows the relationships between these terms.



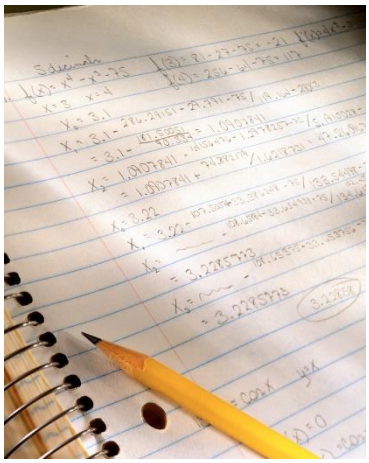
Word Bank

one solution
system of equations
consistent system
dependent
inconsistent system
no solutions
independent
many solutions

Systems of Equations Flowchart
ANSWER KEY



Vocabulary Sheet Teacher Resource



Definition Systems of Equations are two or more interrelated equations involving the same variables. Systems of equations are used when a situation requires the use of two or more variables and two or more equations to model the situation. By using systems of equations, we can solve for more than one variable. A system will have as many equations as there are variables in the system. Systems of equations have three possible outcomes: one solution, many solutions or no solutions. Systems of Equations can be linear or nonlinear.

Classifications of Systems Systems can be classified as consistent or inconsistent and dependent or independent.

Consistent System – A system of equations that has at least one solution.

Inconsistent system – A system of equations that has no solutions

Dependent system – A system in which all the solutions to one equation are also a solution to the other equation.

Independent system – A system with one and only one solution

Strategies for Solving Systems

Graphing – To solve a system with this method, the equations in the system are graphed on the same coordinate graph. Using this method we find the points of intersection in the system. Like any system there are three possible solutions: one solution (a point), many solutions (the line is identical for both equations) and no solutions (the graphs are parallel).

Substitution – To solve a system with this method, substitution is used to reduce two equations with two unknowns to one equation with one unknown. **This method is most useful when one variable can be easily solved for in one of the equations.**

Elimination - This technique is also known as the addition/subtraction or multiplication method. To solve a system with this method, you use addition or subtraction to reduce one equation with two unknowns to one equation with one unknown. **This method is most useful when one variable from both equations has the same coefficient** (the constant a variable is multiplied by) **or the coefficients are multiples of one another.**