# Solving Systems of Linear Equations # Graphing	**Student/Class Goal** Students thinking about continuing their academic studies in a post-secondary institution will need to know and be able to do problems on solving systems of equations.
Outcome (lesson objective) Students will accurately solve a system of equations by graphing.	**Time Frame** 1-2 hours
Standard *Use Math to Solve Problems and Communicate*	**NRS EFL 5-6**

Number Sense	Benchmarks	Geometry & Measurement	Benchmarks	Processes	Benchmarks
Words to numbers connection		Geometric figures		Word problems	5.25, 6.26
Calculation		Coordinate system		Problem solving strategies	
Order of operations		Perimeter/area/volume formulas		Solutions analysis	
Compare/order numbers		Graphing two-dimensional figures		Calculator	5.28, 6.29
Estimation		Measurement relationships		Math terminology/symbols	
Exponents/radical expressions		Pythagorean theorem		Logical progression	
Algebra & Patterns	**Benchmarks**	Measurement applications		Contextual situations	
Patterns/sequences		Measurement conversions		Mathematical material	
Equations/expressions		Rounding		Logical terms	
Linear/nonlinear representations	5.17, 6.17	**Data Analysis & Probability**	**Benchmarks**	Accuracy/precision	
Graphing		Data interpretation		Real-life applications	
Linear equations	5.19, 6.19	Data displays construction		Independence/range/fluency	
Quadratic equations		Central tendency			
		Probabilities			
		Contextual probability			

Materials
Graph paper & colored pencils or dry-erase grid boards and markers
Overhead projector with coordinate grid overlay, overhead markers
How to Change an Equation to the Slope-Intercept Form Resource
How to Graph an Equation in Slope Intercept Form Resource
Sample Systems of Equations Handout
Systems of Equations Activity
Math Scavenger Hunt Teacher Resource
Math Scavenger Hunt Student Answer Sheet
Math Scavenger Hunt Clue Cards
Graphing Scavenger Hunt Information Sheet/Answers

Learner Prior Knowledge
Students should have the ability to solve linear equations, convert linear equations to the slope/intercept form, and graph linear equations in the slope/intercept form. They will have completed earlier lessons on systems of equations, such as *Solving Systems of Linear Equations Introduction.* **Teacher Note** Be sure to classify each system as consistent or inconsistent and dependent or independent.

Instructional Activities
Step 1 - Review how to convert a linear equation/function to the slope intercept form (y = mx + b). (See *How to Change an Equation to the Slope-Intercept Form* Resource) Review how to graph equations using the slope intercept form. (See *How to Graph an Equation in Slope Intercept Form* Resource)

Provide practice with these skills for the students. The samples and practice exercises found in both Resources could be used for student practice or teacher review. Additional practice can be found in math books.

Step 2 - Using the systems of equations found on *Sample Systems of Equations* handout, graph each equation in the system on the same coordinate grid. Use a coordinate grid overhead or large chart paper so students may see the results. Do not pass out the handout sample sheet until after completing the samples together. Be sure to elicit help from the class while graphing the systems of equations.

Include the following questions during instruction: 1) Are the equations in slope-intercept form? 2) Where is the y intercept? 3) What is the slope of the line? 4) What are the coordinates of an additional point on the line?

Complete each example with student assistance. After each example, ask students what the solution to the system of equations is. After completing the examples, the students should recognize the three possible solutions to a system of equations: one solution (where the two graphs intersect at one point), many solutions (when the graphs of both equations are identical) or no solution (when the graphs are parallel to each other).

Step 3 - Distribute the handout, *Sample Systems of Equations,* to the students. Ask students to review the steps to solve each system of equations, graph each system (use the graph found below) and answer the questions (math insights) at the end of the handout.

Step 4 - Students will work independently or in pairs to graph the systems of equations found on the *Systems of Equations* activity. Monitor student understanding by checking student responses as they are working. Based on the needs of the students, provide additional practice from math books.

Step 5 - Set up a Math Scavenger Hunt in the classroom using the directions found in the *Math Scavenger Hunt* Teacher Resource and Answer Sheet. The questions and answers found in the *Graphing Scavenger Hunt* Information Sheet (provide each student with 2 copies) will provide the questions and solutions to place on each clue card. The answers are also included for your reference.

Teacher Note Please be aware that the clue cards provided are smaller than you might want to use in your classroom. They can be enlarged for display or reproduced on large chart paper. Students might have difficulty reading the scale for these graphs as they are based on the lines at .5 intervals.

Assessment/Evidence (*based on outcome)*
Math Scavenger Hunt completed with 100% accuracy.

Teacher Reflection/Lesson Evaluation
This lesson has not yet been field tested.

Next Steps
This is part of a series of lessons on solving systems of linear equations. To continue the study, complete *Solving Systems of Linear Equations Substitutions.* For additional practice, students can work with graphs and equations at Cell Phone Plans.

Technology Integration
Coordinate Grid Paper http://math.about.com/library/Worksheets/Coordinate-Grid-w.pdf
The Math Worksheet Site. Com   http://themathworksheetsite.com/coordinate_plane.html
Webgraphing Tool http://webgraphing.com/multiple_plotting_basic.jsp#
  -- must register, but membership is free
Systems of Linear Equations: Graphing http://www.purplemath.com/modules/systlin2.htm

Purposeful/Transparent
Transitions classes focus on higher level math skills, such as solving systems of equations that will be required for college classes.

Contextual
Students will determine the best method to use when solving systems of equations as they solve problems using graphing, substitution, and elimination/addition.

Building Expertise
Students are given amble practice working in graphing, substitution, and elimination/addition of linear equations and write a system of equations when solving a mathematical situation.

How to Change an Equation to the Slope-Intercept Form **Resource**

Many functions are written in the standard or general form. These equations look something like this:

$$ax + by = c$$

Where a, b, and c are real numbers, and a and b can not both equal zero.

Equations in slope-intercept form look like this:

$$y = mx + b$$

Where m = the slope of the line, b = the y intercept

It is generally easier to graph an equation in the slope-intercept form of an equation. It is possible to convert equation in standard form to slope intercept form.
Study the steps in the example below where a standard equation is changed to slope-intercept form.

Sample 1

2x + y = 18	Original equation
2x -2x +y = -2x + 18	Subtract 2x from both sides to get the y value alone
y = -2x + 18	Check if in slope-intercept form

Sample 2

2x – 3y = 9	Original equation
2x – 2x -3y = -2x +9	Subtract 2x from both sides
-3y/-3 = (-2x +9)/-3	Divide both sides by -3 to eliminate the coefficient with the y.
y = (2/3)x -3	Check if in slope-intercept form

For practice, change the following equations to the slope-intercept form:

$$6x + y = 5$$

$$6 + 2y = 8x$$

$$2y = 6x – 2$$

$$4x – 2y = 12$$

$$9 – 3y = 3x$$

$$2y – 4x + 6 = 0$$

$$-2y = 4x + 5$$

$$3x + y = 8$$

An answer sheet can be found following resource

Resource Answer Sheet

$6x + y = 5$	$y = -6x + 5$
$6 + 2y = 8x$	$y = 4x - 3$
$2y = 6x - 2$	$y = 3x - 1$
$4x - 2y = 12$	$y = 2x - 6$
$9 - 3y = 3x$	$y = -x + 3$
$2y - 4x + 6 = 0$	$y = 2x - 3$
$-2y = 4x + 5$	$y = -2x - 5/2$
$3x + y = 8$	$y = -3x + 8$

How to Graph an Equation in Slope Intercept Form Resource

$$y = mx+b$$

In the slope intercept form of a linear equation, each letter represents a special value.
m = the slope of the equation
b = the y intercept

With these two facts, an equation in slope intercept form can be graphed.

With these facts, let's look at some equations:

Sample 1   y = 4x + 2
Sample 2   y = -x + 6
Sample 3   y = 2x -4
Sample 4   y = 1/4x

First locate point b on the graph and mark it.
Since this is the point where the graph of the equation crosses the y-axis, the x value will be zero and the y value will be the b value in the equation (0, b).

The coordinates of the y intercept in the samples would be:
Sample 1   (0,2)
Sample 2   (0,6)
Sample 3   (0,-4)
Sample 4   (0,0)

Second, determine the slope (m) of the equation. What number is before the x in the equation? If there is not a numeral in front of the letter x, remember the number 1 is understood.

The slopes of the equations in the samples are:
Sample 1   4
Sample 2   -1
Sample 3   2
Sample 4   ¼

Slope is the ratio of the change in the y value of 2 points (y2-y1) over the change in the x value of the same two points (x2-x1). This is commonly known as the rise/run. The slopes of the samples could be written as 4/1, -1/1, 2/1 and ¼ respectively. Using these values, plot a second point for each equation.

For example 1, start at the y intercept (0, 2). From this point, use the slope of the equation to find a second point on the line. The slope of the line is 4/1, so count up four spaces (the rise) and count to the right one space (the run). Where you end up is a second point on the graph (1, 6). Repeat theses steps to find a third point on the graph. Remember, the slope of the line will determine the direction of the rise and run. Going up or to the right are positive directions. Going left or down are negative

directions. If the slope is negative, you must have **one** movement in a negative direction – not both. (Remember a negative divided by a negative is a positive.)

Note Positive slopes will start lower on the left side of the graph and end up higher on the right side of the graph. Lines with negative slopes will start higher on the left side and end up lower on the right side.

The graphs of the four sample exercise are below.

Sample 1
$y = 4x + 2$

Sample 2
$y = -x + 6$

Sample 3
$y = 2x - 4$
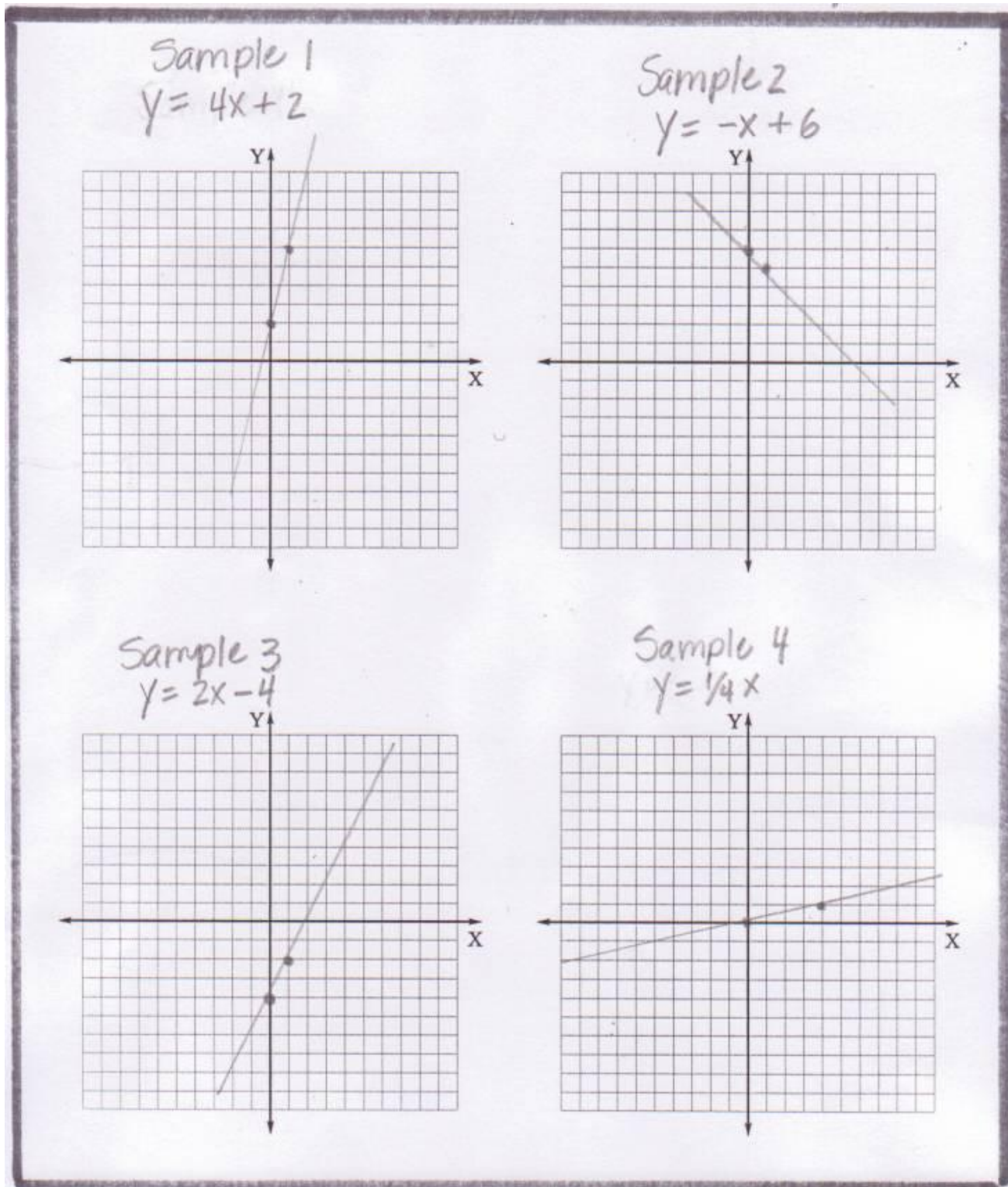
Sample 4
$y = \frac{1}{4}x$

Sample Systems of Equations

Study each system of equations and the solution shown below:

Sample 1

$$3x - 2y = 6$$
$$x + y = 2$$

Change both equations to slope intercept form:

$3x - 2y = 6$	$x + y = 2$
$-2y = -3x + 6$	$y = -x + 2$
$y = (3/2)x - 3$	
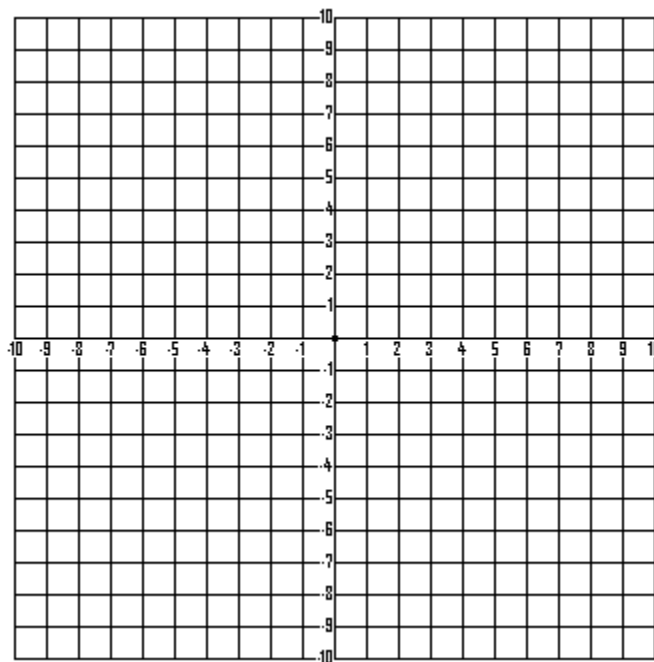
Sample 2

$$2x - y = 1$$
$$4x - 2y = 2$$

Change both equations to slope intercept form:

$2x - y = 1$	$4x - 2y = 2$
$-y = -2x + 1$	$-2y = -4x + 2$
$y = 2x - 1$	$y = 2x - 1$

Sample Systems of Equations Handout
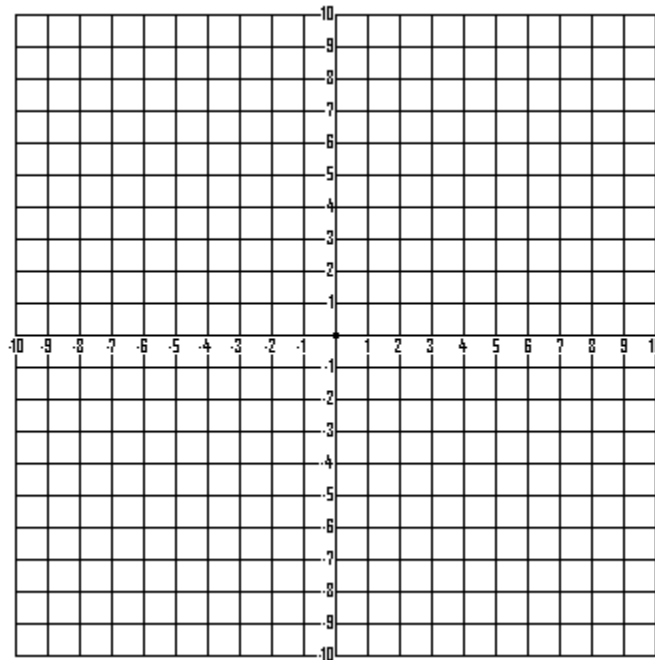
Sample 3

$$y = 2x + 1$$
$$y = 2x - 3$$

Note Since both equations in sample 3 are in the slope intercept form, they can be graphed without additional changes.

Sample Systems of Equations Handout
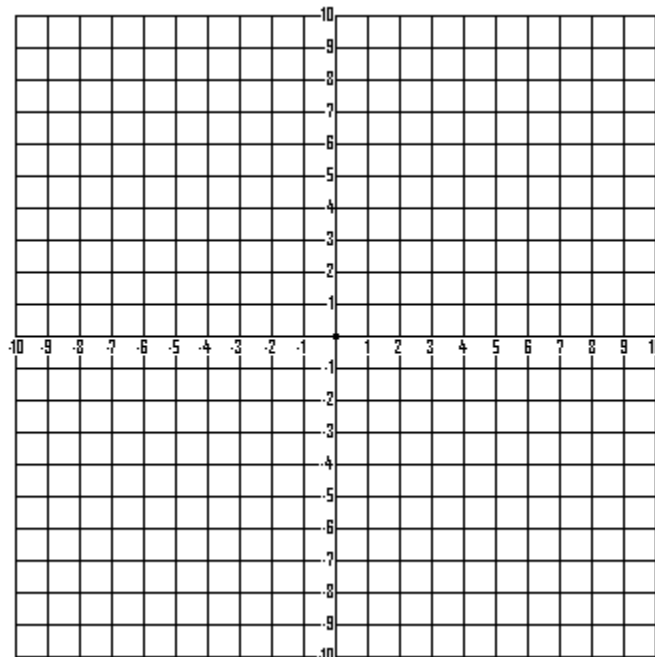
Math Insights

1. What is the solution to Sample 1? (Give the coordinates of the solution.)

2. What is the solution to the system in sample 2?

3. Can two equations that appear different (Sample 2) actually be identical?
   - ☐ Yes
   - ☐ No

4. If two equations have the same slope (Sample 3), what is true about their solution?

5. How many different types of solutions are there for a system of equations?

6. Which systems are consistent?

   Which systems are inconsistent?

   Which systems are dependent?

   Which systems are independent?

Sample Systems of Equations
ANSWER KEY
Note: The scale of the two axes is slightly different. Your answers may look slightly different.

Sample 1

$$3x - 2y = 6$$
$$x + y = 2$$
Intersection (2,0)
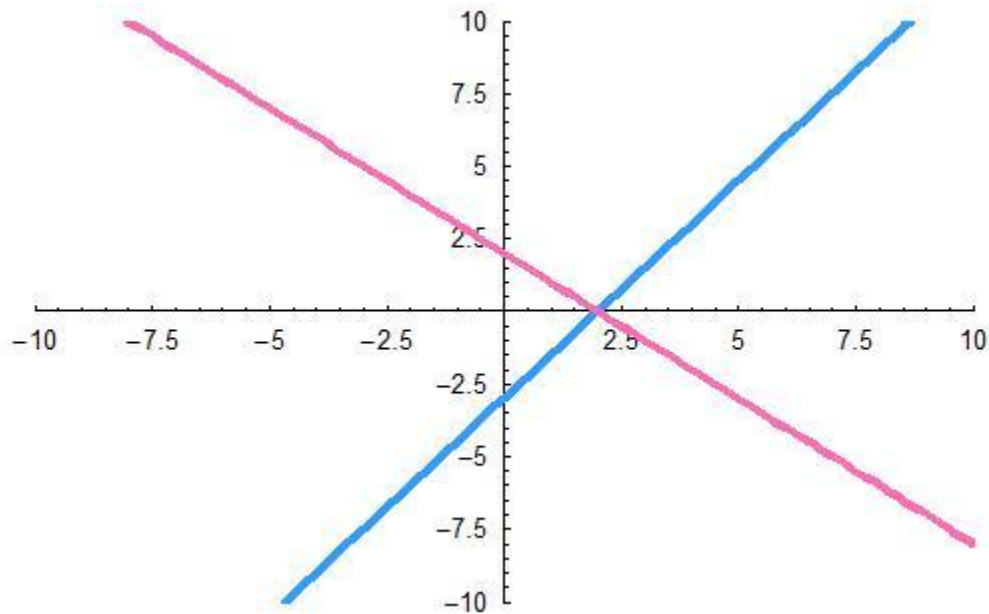
Sample 2

$$2x - y = 1$$
$$4x - 2y = 2$$
Many Solutions
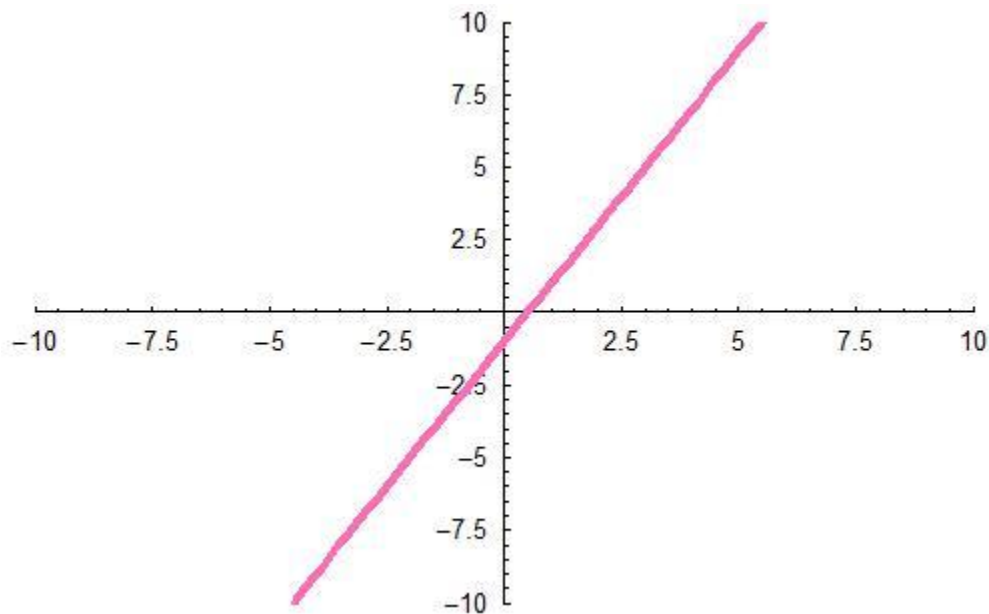
Sample Systems of Equations
ANSWER KEY
Note: The scale of the two axes is slightly different. Your answers may look slightly different.

Sample 3

$$y = 2x + 1$$
$$y = 2x - 3$$
No Points of Intersection

.
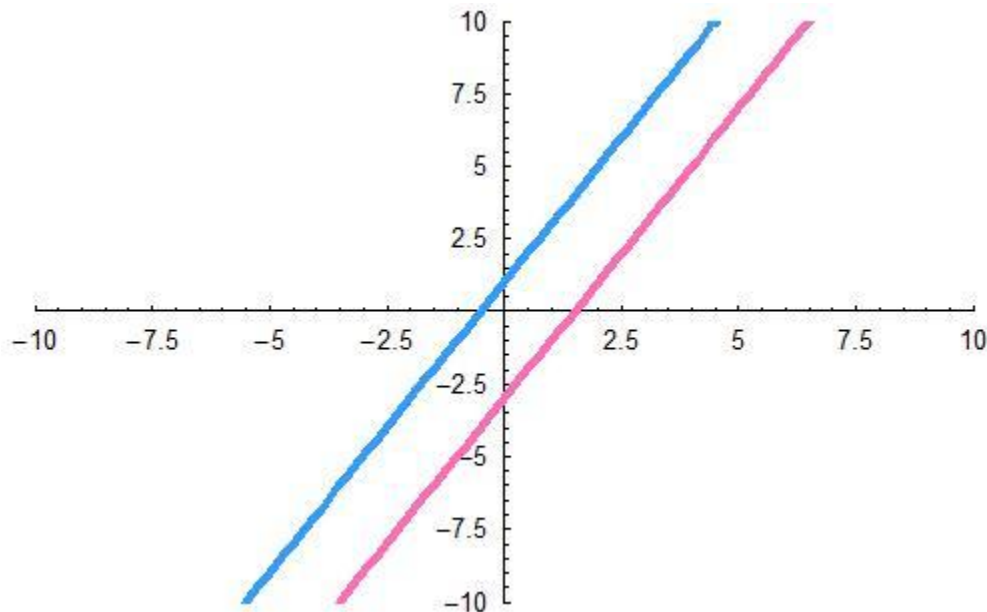
Math Insights

1. What is the solution to Sample 1? (Give the coordinates of the solution.)
   (2,0) x = 2, y = 0

2. What is the solution to the system in sample 2?
   All solutions

3. Can two equations that appear different (Sample 2) actually be identical?
   Yes - One of the equations can be simplified

4. If two equations have the same slope (Sample 3), what is true about their solution?
   There is no solution   **Note: y-intercept is different in each equation**

5. How many different types of solutions are there for a system of equations?
   one solution, consistent, independent
   many solutions, consistent, dependent
   no solutions, inconsistent

6. Which systems are consistent? **Sample 1, 2** Which systems are inconsistent? **Sample 3** Which systems are dependent? **Sample 2** Which systems are independent? **Sample 1**

Systems of Equations Activity

Graph the following systems of equations to find the solution to each system.
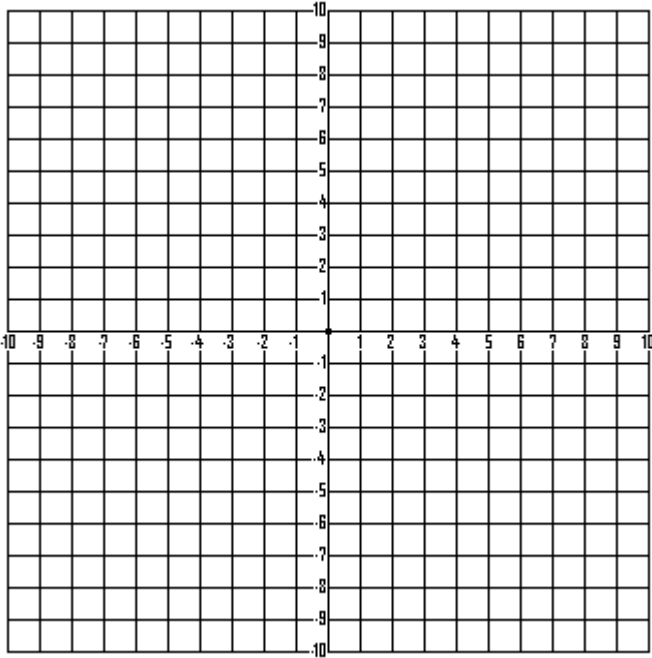
$$y = -4x + 5$$
$$y = 3x - 9$$

$$y = -3x + 7$$
$$y = 2x - 3$$

$$x + 3y = 6$$
$$x - 3y = 6$$

$$y = x + 6$$
$$y = -2x$$
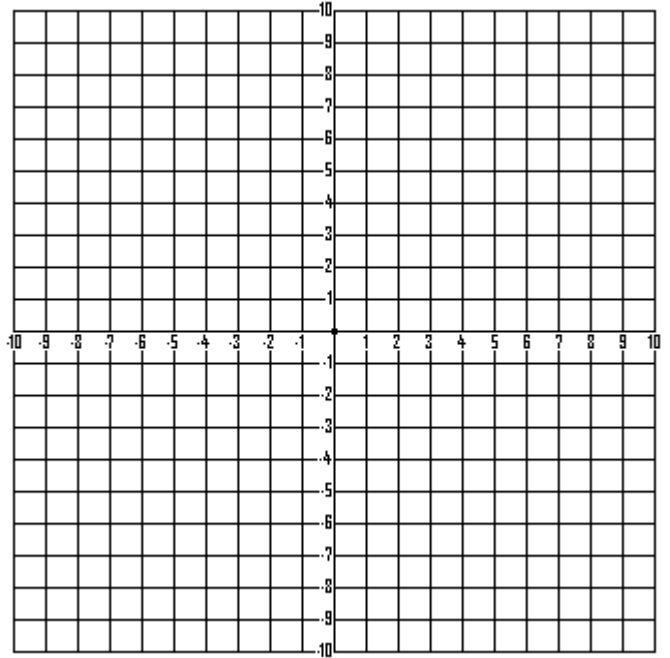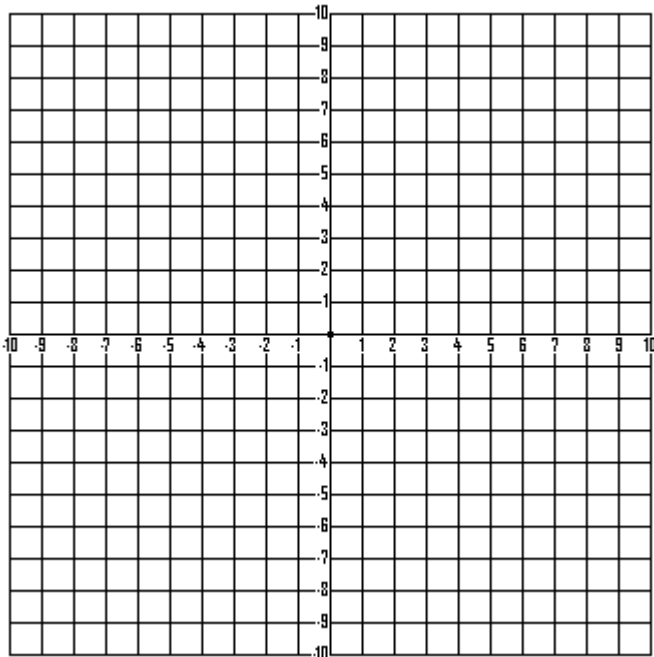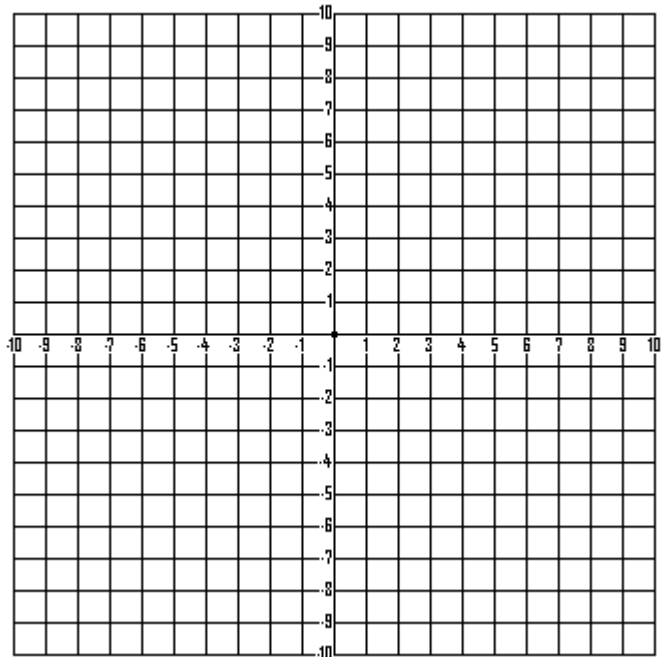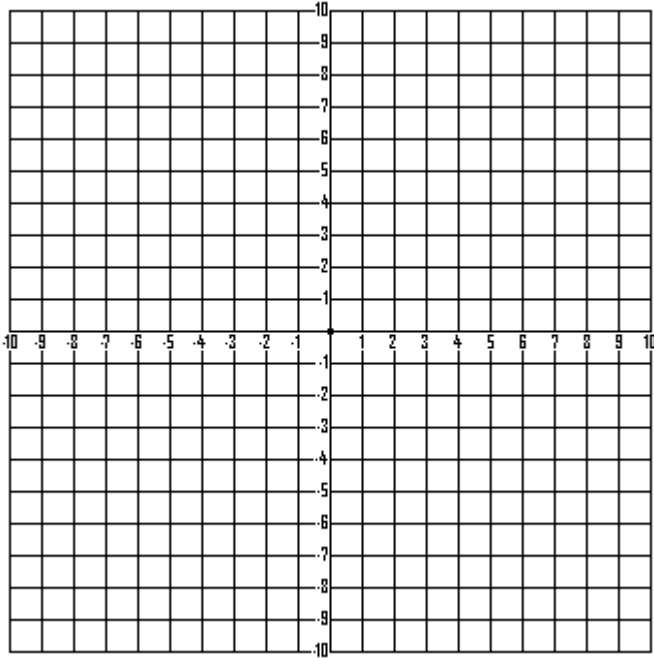
$$y = 4x - 3$$
$$y = -2x + 9$$

$$3x - 2y = 4$$
$$y = -2x + 5$$
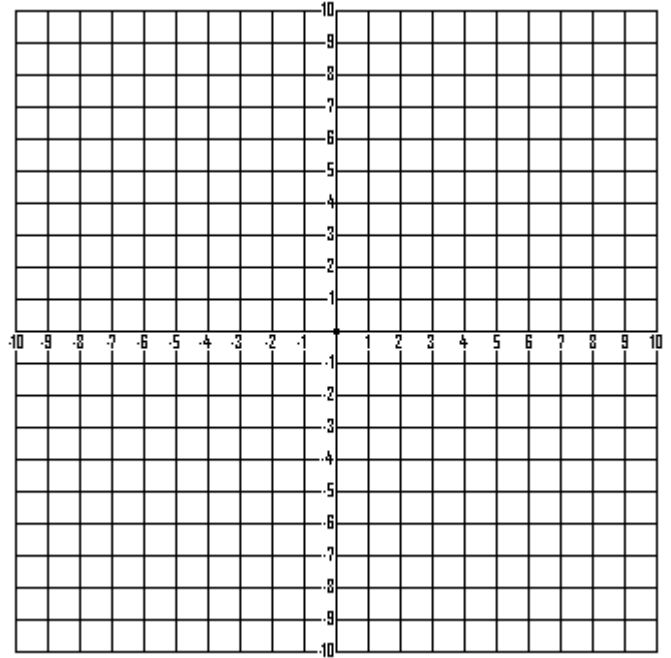
$$3x - 2y = 6$$
$$x - y = 2$$
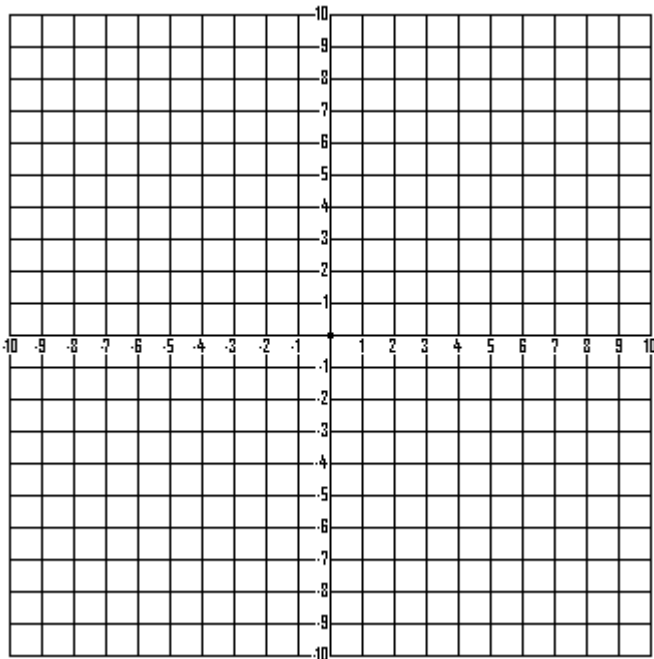
$$x + y = 4$$
$$2x + 2y = 10$$

Systems of Equations Activity
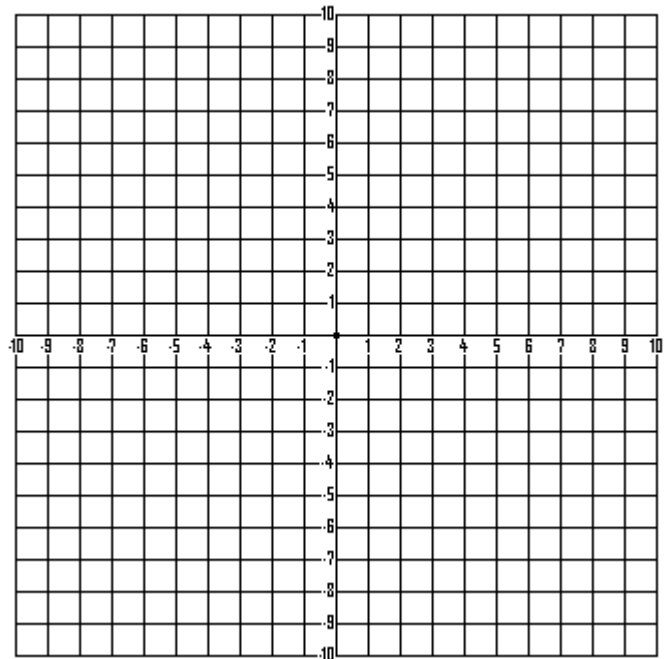
Systems of Equations Activity
ANSWER KEY
Note: Scale is slightly different for the answer key.

y = -4x + 5
y = 3x − 9
Intersection (2,-3)
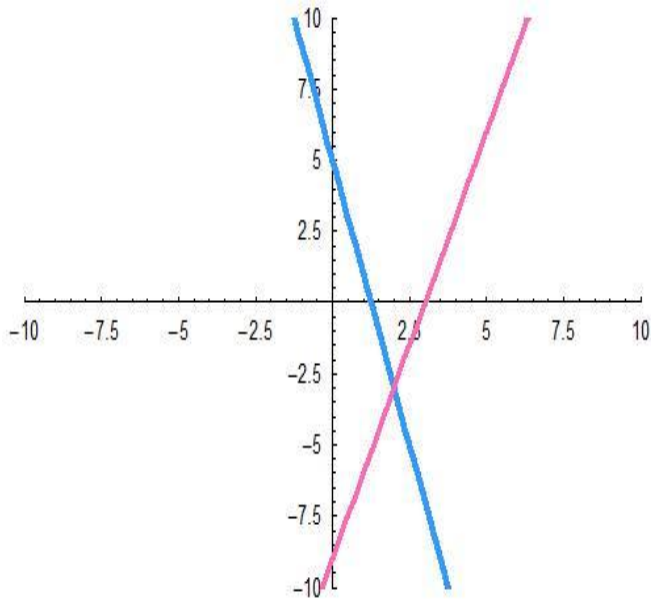
y = -3x + 7
y = 2x − 3
Intersection (2,1)
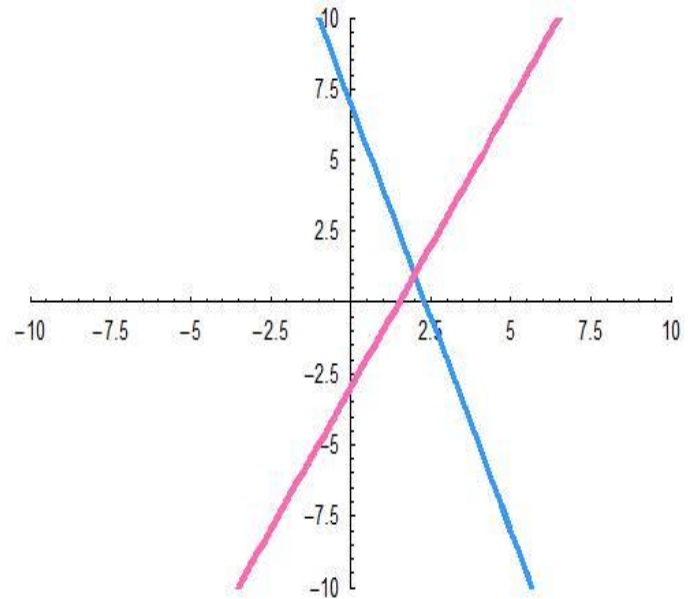
x + 3y = 6
x − 3y = 6
Intersection (6,0)
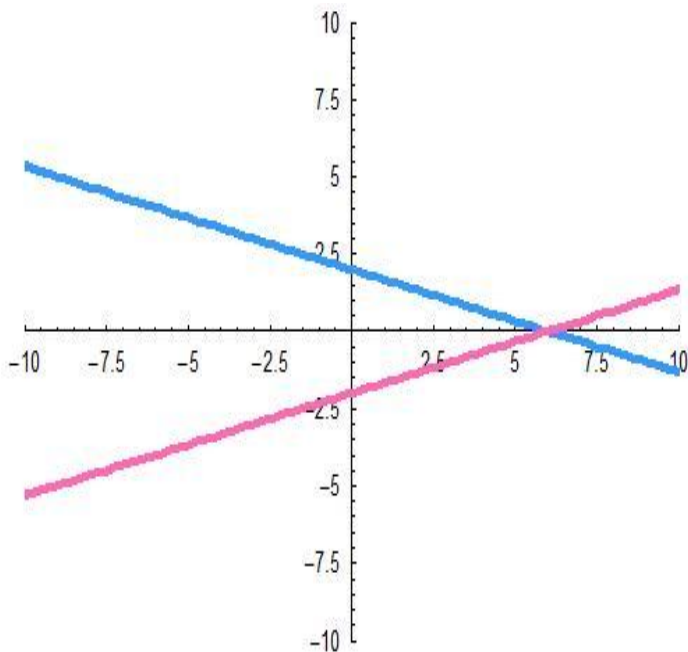
y = x + 6
y = -2x
Intersection (-2,4)
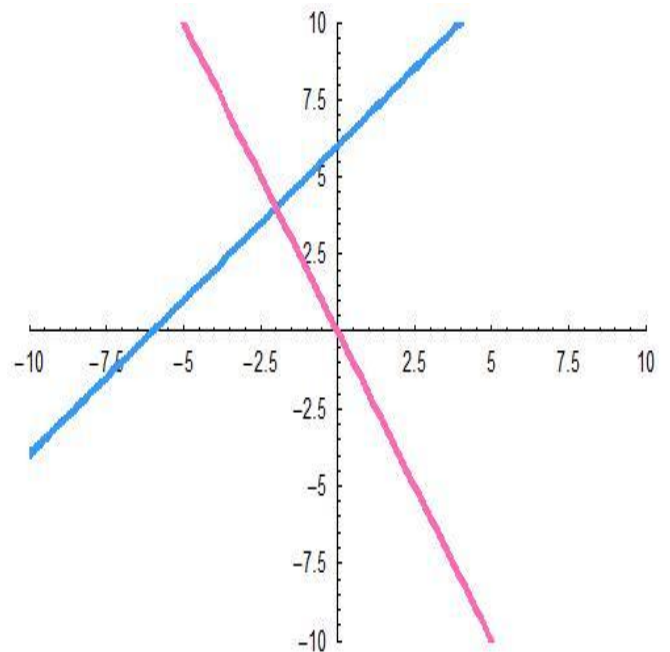
y = 4x − 3
y = -2x + 9
Intersection (2,5)
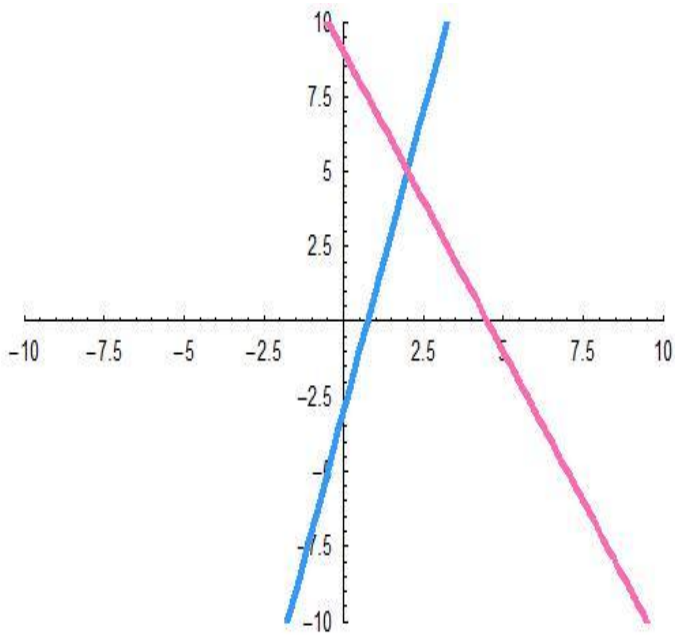
3x − 2y = 4
y = -2x + 5
Intersection (2,1)
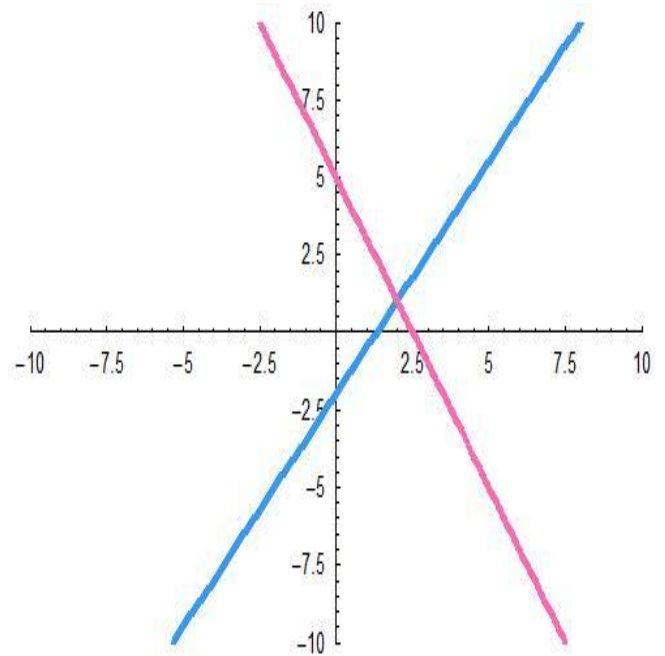
3x − 2y = 6
x − y = 2
Intersection (2,0)
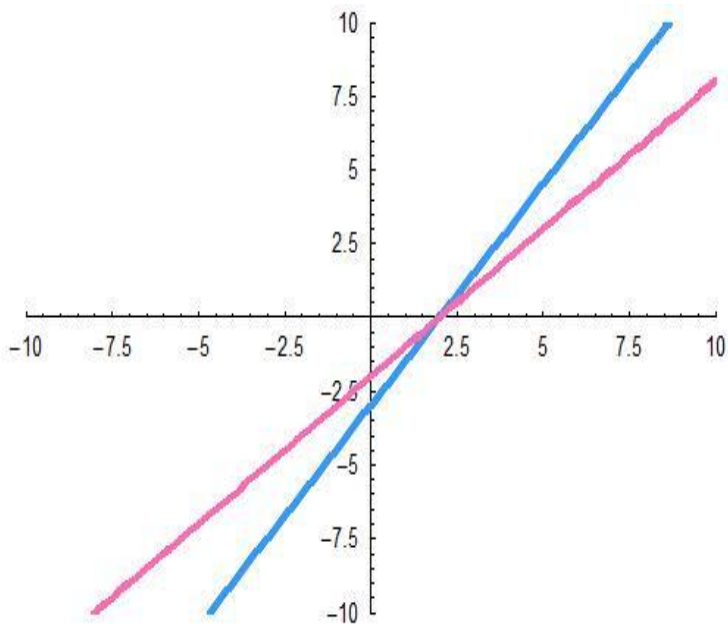
x + y = 4
2x + 2y = 10
No Solution
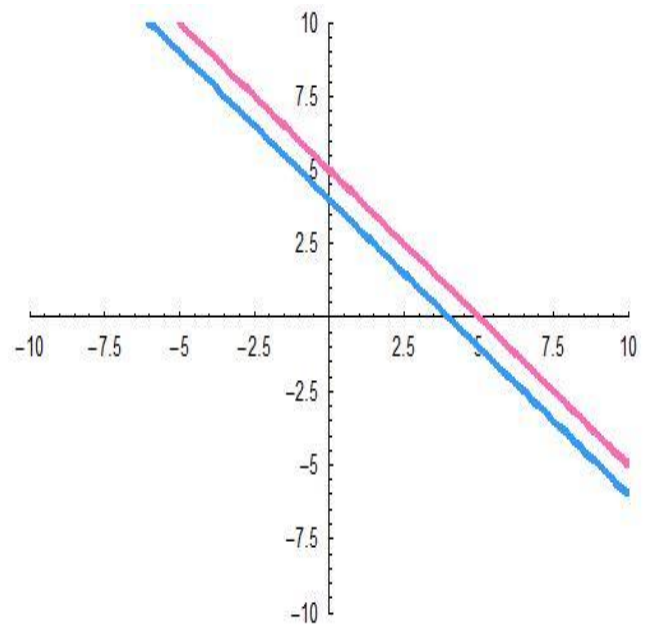
Math Scavenger Hunt Teacher Resource

Supplies
Paper or card stock (8 ½" by 11")
2 colored markers
Tape

A math scavenger hunt is a fun way to assess the math skills of your students. Most any math topic can be evaluated with this activity, and the students will stay active as they move around the room solving problems and searching for the answers. Students can work in groups or alone as they complete the activity.

To set up a scavenger hunt select 6-8 problems with answers. Before you make the scavenger hunt clue cards, do some planning to make sure each problem and its answer will be on different cards. This has already been done for you in the series of lessons on systems of equations. When you have decided on the problem and answer to place on each card, write a problem at the top (portrait orientation) of the clue card and a solution at the bottom of the card. Write all the answers in one color of marker, and use the second color for the problems. Tape these sheets around the room.

Math Clue Card Example

2 x 4
10

Now it is time for the students to complete the Math Scavenger Hunt. Give each student a Scavenger Hunt Answer Sheet (see below). Students can start their hunt at any location in the room. This way the class will be spread out around the classroom. At their first stop, the students will write the problem on their answer sheet and solve it. Remember the problem will be at the bottom of the sheet. There is space on the answer sheet for the students to show their work. Once they have solved this problem they will find the Scavenger Hunt Clue Card with their answer. The problem at the bottom of this clue card will be the students' next problem to solve. If the students don't find their answer when they look around the room, the students know to redo their work. Students continue with this process until all the problems have been completed, and they return to the card which contains their first problem.

The answers can be corrected quickly because the answers will be in a specific order. Remember each student will start the Scavenger Hunt in a different place in the answer sequence.
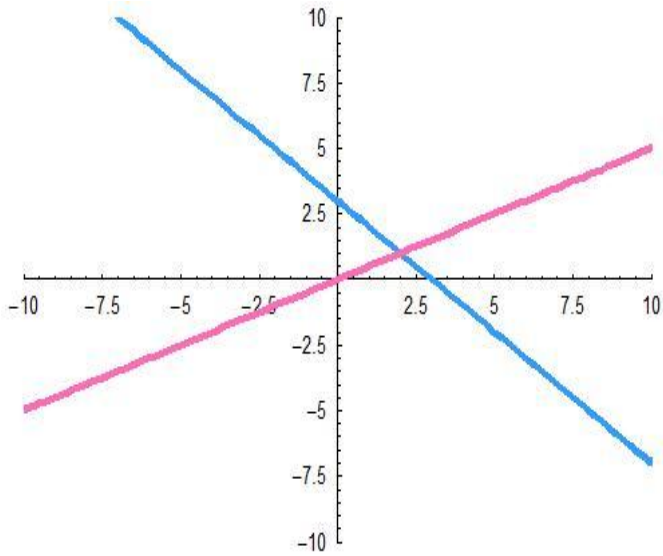
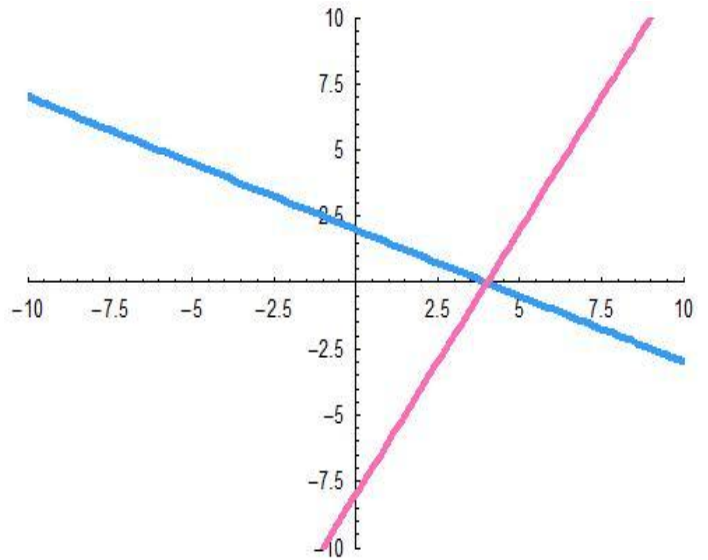# Math Scavenger Hunt Student Answer Sheet
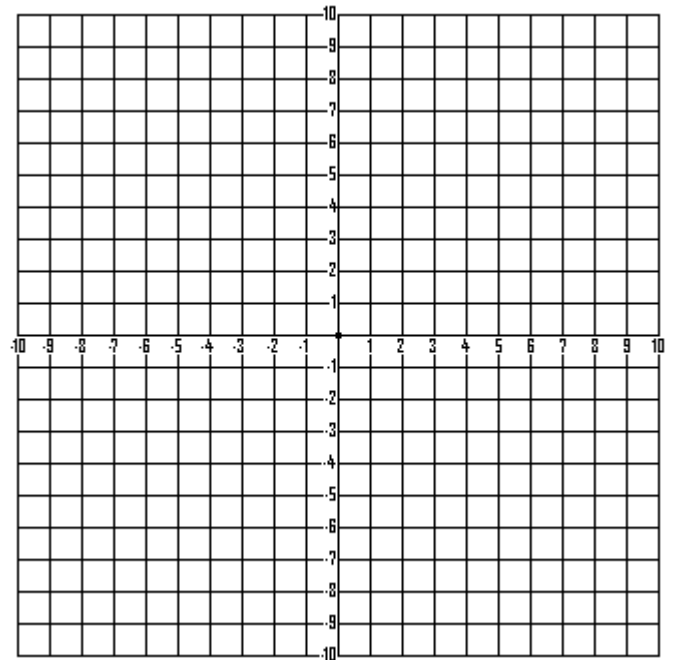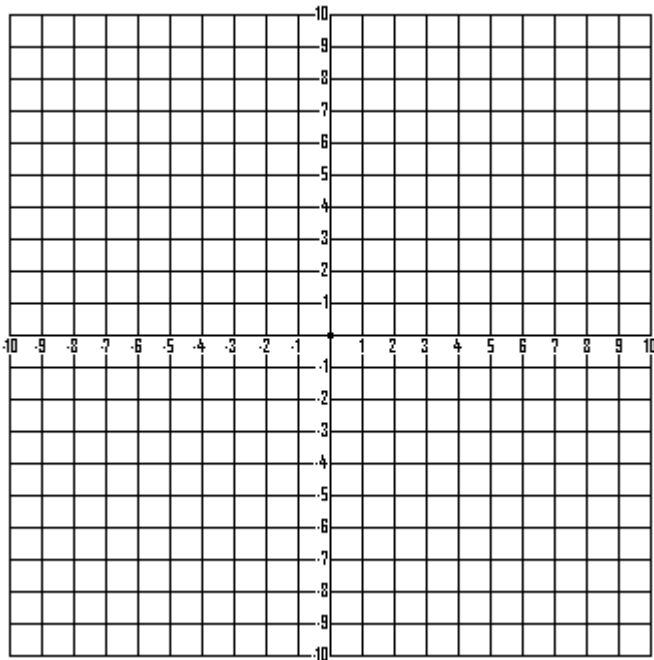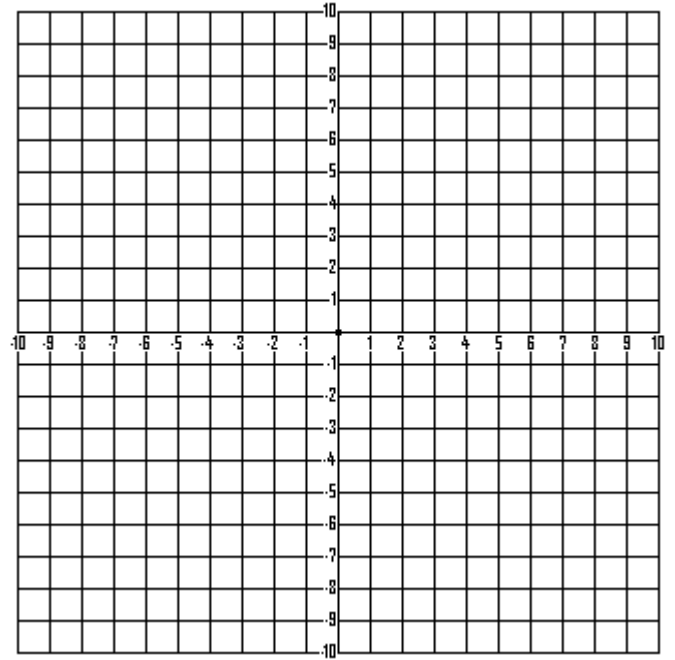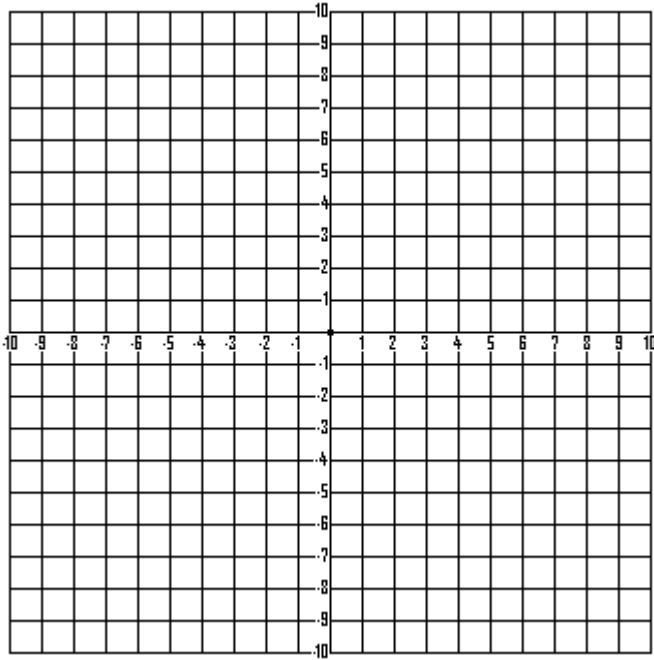
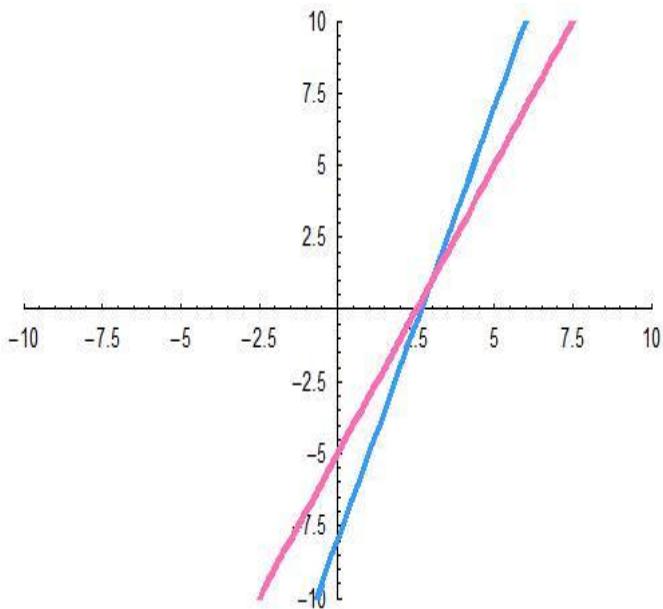# Graphing Scavenger Hunt Information Sheet
Math Clue Cards

$y = -2x + 3$
$y = 1/2x + 3$

$y - 3x = -8$
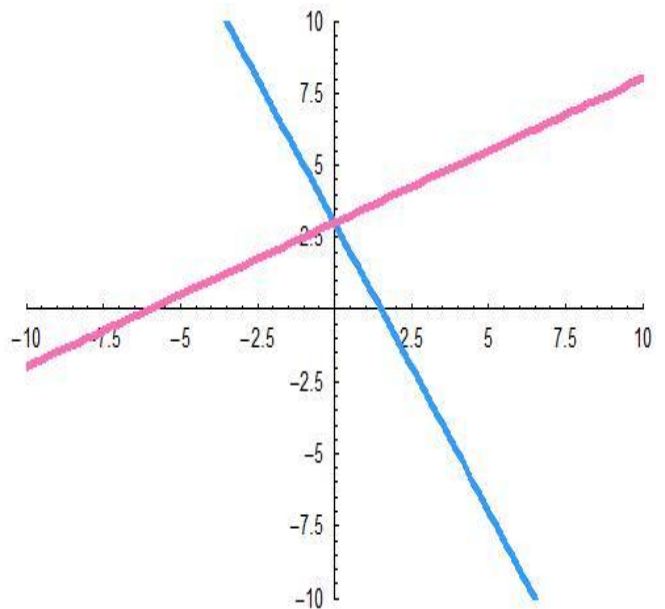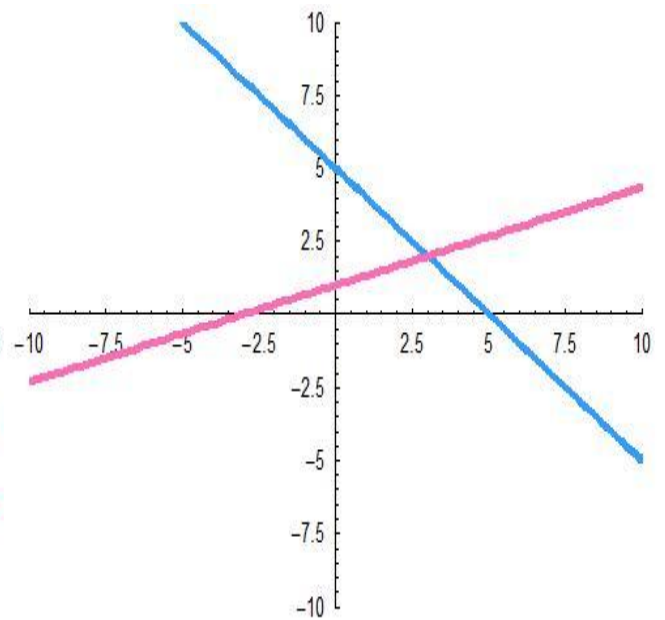$y + 5 = 2x$

$y = -x + 3$
$y = 1/2 x$

$y = -x + 5$
$y = 1/3 x + 1$

Graphing Scavenger Hunt Information Sheet
Math Clue Cards

$x + 2y = 4$
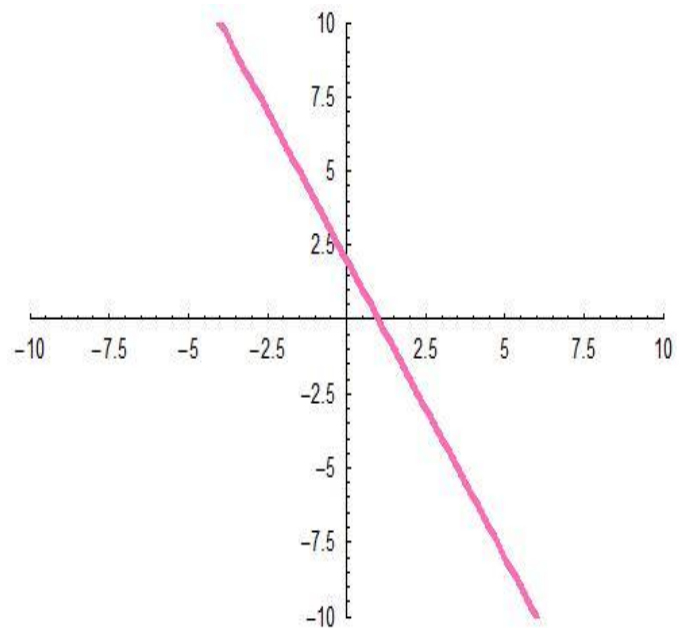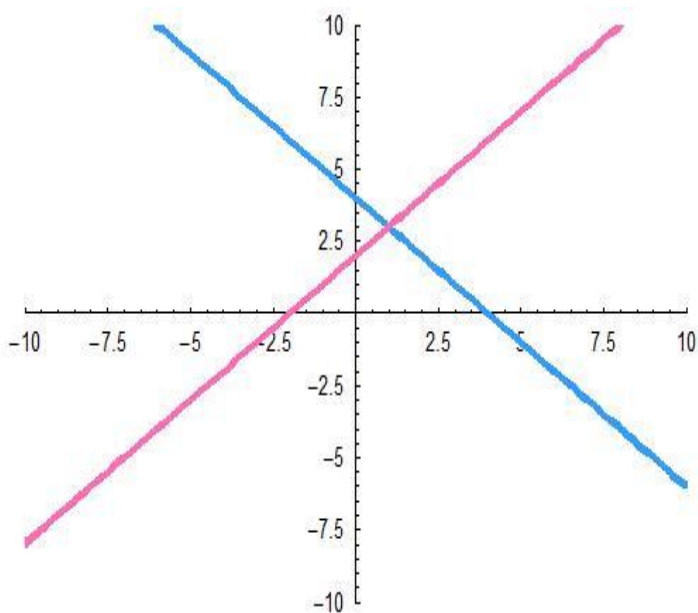$2x - y = 8$

$x + y = 4$
$y - x = 2$

$3x + y = 3$
$x - 4y = -12$

$y = -x + 6$
$y = -x + 3$
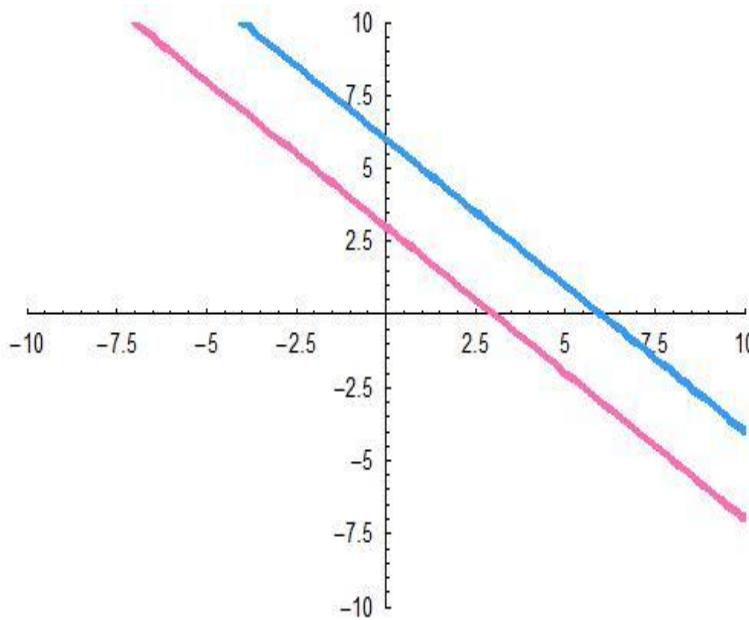
Graphing Scavenger Hunt Information Sheet

Math Clue Cards ANSWER KEY
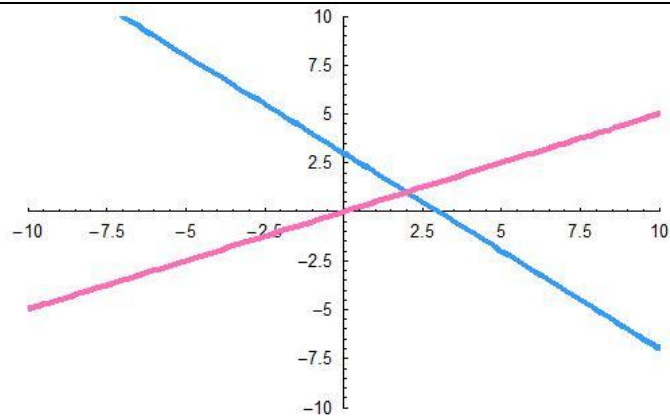
1 $y = -x + 3$ $y = 1/2\ x$	
2 $x + 2y = 4$ $2x - y = 8$	
3 $y - 3x = -8$ $y + 5 = 2x$	
4 $y = -2x + 3$ $y = 1/2x + 3$	

5	$y = -x + 6$ $y = -x + 3$	
6	$y = -x + 5$ $y = 1/3 \, x + 1$	
7	$x + y = 4$ $y - x = 2$	
8	$3x + y = 3$ $x - 4y = -12$	