Simplify Rational Expressions

Student/Class Goal
As students prepare for postsecondary courses in algebra, they must become proficient simplifying rational expressions.

Outcome *(lesson objective)*
Students will simplify rational expressions with polynomials and find the greatest common factor (GCF).

Time Frame
4 hours

Standard *Use Math to Solve Problems and Communicate*

NRS EFL 6

Number Sense	Benchmarks	Geometry & Measurement	Benchmarks	Processes	Benchmarks
Words to numbers connection		Geometric figures		Word problems	
Calculation	6.1	Coordinate system		Problem solving strategies	
Order of operations		Perimeter/area/volume formulas		Solutions analysis	
Compare/order numbers		Graphing two-dimensional figures		Calculator	
Estimation		Measurement relationships		Math terminology/symbols	
Exponents/radical expressions	6.5	Pythagorean theorem		Logical progression	
Algebra & Patterns	**Benchmarks**	Measurement applications		Contextual situations	
Patterns/sequences		Measurement conversions		Mathematical material	
Equations/expressions	3.15, 4.16, 5.16, 6.16	Rounding		Logical terms	
Linear/nonlinear representations		**Data Analysis & Probability**	**Benchmarks**	Accuracy/precision	
Graphing		Data interpretation		Real-life applications	
Linear equations		Data displays construction		Independence/range/fluency	
Quadratic equations		Central tendency			
		Probabilities			
		Contextual probability			

Materials
white board/chalk board
pencil and paper
calculators

Learner Prior Knowledge
Students should have an understanding of the Distributive Property, solving equations with one unknown and the Addition and Multiplication Properties of Equality, and factoring polynomials.

Instructional Activities
Teacher Note In all activities, first demonstrate to students some examples, then have the students help you solve them. Finally, have students solve examples on their own or in pairs. Handouts can be made prior to class from the examples listed below, websites, or classroom resources.

Step 1 - Explain to students that simplifying an algebraic expression means we write it in the most compact or efficient manner, without changing the value of the expression. This involves collecting like terms, which means that we add together anything that can be added together. The rule here is that only *like* terms can be added together. $5x$, x, and $-3x$ are like terms as are $4x^2$ and $-2x^2$. Terms with exponents such as xy^2, $5y^2x$, and $7xy^2$ are also like terms. Terms xy^2 and x^2y are **NOT** like terms because the same variable is not raised to the same power. Practice with students on combining like terms.

Examples
a) $5x^2 + 6x^2 = (5 + 6)x^2$ is $11x^2$
b) $x^2 + 5x + 7x^2 + 3 + 2x + 9$ is $8x^2 + 7x + 12$
c) $3y + 2 + 7y - 8 + 2y^2$ is $2y^2 + 10y - 6$
d) $7y^3 + 6y + 8 + 9y^2 - 5y - 3y^3 + 9$ is $4y^3 + 9y^2 + y - 1$

Step 2 - Show how the Distributive Property allows us to combine like terms. Practice using the Distributive Property with students.

Examples a) $7(n + 5)$ is $7n + 37$
b) $2(n - p)$ is $2n - 2p$
c) $5(7 - 4y)$ is $35 - 20y$
d) $6(a - 7)$ is $6a - 42$

Step 3 - Discuss how expressions with parentheses must be multiplied out before collecting like terms. You cannot combine terms in parentheses (or other grouping symbols) with terms outside the parentheses. Think of parentheses as walls — the terms inside the parentheses can't "see" the terms outside the parentheses. If there is some factor multiplying the parentheses, then the only way to get rid of the parentheses is to multiply using the distributive law.

Example $5n + 6(n - 2)$ is $5n + 6n - 12$ is $11n - 12$

Practice simplifying expressions with parentheses with students.

Examples a) $3n - 2(n + 4)$ is $3n - 2n - 6$ is $n - 6$
b) $4y^2 + 5y + 10(y^2 - y - 3)$ is $4y^2 + 5y + 10y^2 - 10y - 30$ is $14y^2 - 5y - 30$
c) $6(r + 4) - 8(r - 3)$ is $6r + 24 - 8r + 24$ is $-2r + 48$
d) $n(n + 3) + 5(n - 2)$ is $n^2 + 3n + 5n - 10$ is $n^2 + 8n - 10$

Step 4 - Show how a negative sign in front of parentheses changes the signs inside the parentheses. When a minus sign appears in front of parentheses it may seem as though there is no factor multiplying the parentheses. Instruct students that the minus sign indicating subtraction should always be thought of as adding the opposite. This means that you want to add the opposite of the entire thing inside the parentheses, and so you have to change the sign of each term in the parentheses. In other words, a negative sign in front of parentheses implies a factor of one in front of the parentheses. The minus sign makes that factor into a negative one, which can be multiplied by the distributive law.

Example $5x - (8 - 2x)$
$= 5x + (-1)(8 - 2x)$
$= 5x - 8 + 2x$
$= 7x - 8$

It is helpful to always think of minus signs as being "stuck" to the terms directly to their right. That way, as you rearrange terms, collect like terms, and clear parentheses, the minus signs will go with whatever was to their right. If what is immediately to the right of a minus sign happens to be a parenthesis, then the minus sign changes every term inside the parentheses.

Practice several problems like this with students before continuing.

Examples a) $7n - (4 + 3n)$ is $7n - 4 - 3n$ is $4n - 4$
b) $4(n - 2) - (n + 4)$ is $4n - 8 - n - 4$ is $3n - 12$
c) $6(x + 2) - (4x - 2)$ is $6x + 12 - 4x + 2$ is $2x + 14$
d) $10n - (3n - 4) - (2 + n)$ is $10n - 3n + 4 - 2 - n$ is $6n + 2$

Step 5 - Explain to students that rational expressions or algebraic fractions are algebraic expressions whose numerator and denominator are polynomials. Examples of these are:

$$\frac{9}{x} \qquad \frac{10n + 3}{y - 2} \qquad \frac{a - 5}{a^2 + 8}$$

Zero cannot be used as a denominator because division by zero is undefined. Therefore, any value assigned to a variable that results in a denominator of zero must be excluded from the domain of the variable. In the above examples, $x \neq 0$ and $y \neq 2$

Demonstrate to students how to simplify an algebraic fraction.

Example Simplify the following expression. $\dfrac{18\,a^2\,b\,c}{36\,a\,b^3\,c^2}$

Step 1: Factor the numerator = 2 x 3 x 3 x a x a x b x c

Step 2: Factor the denominator = 2 x 2 x 3 x 3 x a x b x b x b x c x c

Step 3: Simplify the common factors, note that a, b or c cannot equal 0

$$\frac{\cancel{2} \times \cancel{3} \times \cancel{3} \times \cancel{a} \times a \times \cancel{b} \times \cancel{c}}{\cancel{2} \times 2 \times \cancel{3} \times \cancel{3} \times \cancel{a} \times \cancel{b} \times b \times b \times \cancel{c} \times c} \quad \text{this equals} \quad \frac{a}{2\, b^2\, c}$$

Example Simplify the following expression. $\frac{24\, a^3\, a^2\, b\, c^2}{15\, a\, b^3\, c^2}$

$$\frac{2 \times 2 \times 2 \times \cancel{3} \times \cancel{a} \times a \times a \times a \times a \times \cancel{b} \times \cancel{c} \times \cancel{c}}{\cancel{3} \times 5 \times \cancel{a} \times \cancel{b} \times b \times b \times \cancel{c} \times \cancel{c}} \quad \text{this equals} \quad \frac{8\, a^4}{5\, b^2}$$

Step 6 - Show students how to find the Greatest Common Factor (GCF). For a number, the Greatest Common Factor (GCF) is the largest number that will divide evenly into that number. For example, for 48, the GCF is 24. For a *polynomial*, the GCF is the largest monomial that divides (is a factor of) each term of the polynomial.

Example Find the GCF of the list of monomials: n^7, n^4, n^3
The exponents on the n's are 7, 4 and 3. We can only divide out all the terms by the one monomial n^3. The GCF = n^3

If all terms have the same variable, the GCF for the variable part is that variable raised to the lowest exponent that is listed.

Example Find the GCF of the list of monomials: $3xy^3$, $9x^2y^2$, $18x^3y$

First look at the numerical part. We have a 3, 9 and 18. The largest number that can be divided out of those numbers is 3 which means the numerical GCF is 3.

Now do the variable part. It looks like each term has an *x* and a *y*. In both cases the lowest exponent is 1 which means the GCF of the variable part is xy. Put the two together and we have a GCF of 3*xy*.

Example Factor out the GCF: $8x^3 + 4x^2 + 2x$

Step 1: Identify the GCF. We have 2, 4 and 8. The largest numerical factor is 2. We have x^3, x^2 and x. We can only divide each by x. The GCF = 2x

Step 2: Divide the GCF out of every term of the polynomial.

$$\frac{8x^3 + 4x^2 + 2x}{2x} = 2x(4x^2 + 2x + 1)$$

Explain to students that if a term of the polynomial is exactly the same as the GCF, when you divide it by the GCF you are left with 1, not 0. As shown above, when we divide 2*x* by 2*x* we get 1, so we need a 1 as the third term inside of the ( ).

Demonstrate to the students that if we multiply our answer out, we get the original polynomial. Factoring gives you another way to write the expression so it will be equivalent to the original problem.

Choose one of these real-life applications for students to complete: Simplification of Rational Expressions Applications or Applications of Rational Expressions.

Assessment/Evidence (*based on outcome)*
Problems and real-life applications

Teacher Reflection/Lesson Evaluation
This lesson has not yet been field tested.

Next Steps
For additional practice, students can work the problems at Simplifying Rational Expressions or Rational Expressions.

Technology Integration
Rational Expressions: Simplifying http://www.purplemath.com/modules/rtnldefs2.htm
Simplifying Rational Expressions http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut8_simrat.htm
Simplifying Rational Expressions http://www.khanacademy.org/video/simplifying-rational-expressions-2?playlist=Algebra%20I%20Worked%20Examples

Purposeful/Transparent
As part of a postsecondary preparation algebra course, students work on problems to become proficient in simplifying rational expressions and equations.

Contextual
Students master algebraic concepts and then solve real-life problems dealing with the concept of rational expressions.

Building Expertise
Building on student's understanding of the distributive property, the class practices simplifying rational expressions, considering the effect of the minus sign in an equation, the algebraic fraction, and the greatest common factor.