
mathematician. Ask what math concepts quilters need and make note of students' responses on the board or chart paper.

Step 2 - Since every quilter is a mathematician, understanding how shapes fit together to form other shapes requires fundamentals of geometry. Visualizing how those shapes can form a balanced pattern means understanding symmetry in design, an important mathematical concept.

Give students an opportunity to use manipulatives to examine quilts as mathematicians looking for common aspects - shapes, colors, patterns - in the quilts they see. The Teacher Resource Improving Mathematics Teaching by Using Manipulatives provides background information and resources that every teacher should have in their ABLE classroom. Share pictures of quilts from books or actual quilts. Choose from these options based on student's levels:

Pattern Tiles or Blocks Give students several colors of tiles and allow them to create patterns. What patterns can they make? What are the diagonal and vertical patterns? Encourage using three or four colors as they build their designs. Explore combinations that radiate from center. They can also record their favorite patterns on grid paper. Using 3 colors, how many different combinations can they make for a nine-patch square ( 3 tiles by 3 tiles)?

Geoboards and rubber bands can be used to form geometrical shapes. Provide samples of black and white instructional quilt diagrams used by quilters to fit pieces together. Good sources are quilting pattern magazines or how-to books. A Google search for "quilt patterns" will provide multiple sites where patterns can be found. Select patterns and use colored rubber bands to re-create patterns/diagrams or construct original patterns. Shapes can be used to form other shapes -2 triangles form a square, 2 rhombuses form a hexagon.

Quilt Blocks Prior to class, iron assorted calico fabrics onto tagboard using fusible webbing (available at fabric stores). Using the pattern of 4-inch squares, cut tagboard fabric into square and triangle (half-square) shapes. Have the students mix and match into combinations.

Step 3 - Discuss the nature of quilts and how the angles make it possible for the patterns to tessellate the plane. Tessellation refers to small squares or blocks, as floors or pavements form or arrange in a checkered or mosaic pattern; identical shapes fit together, such as triangles tessellate.

Using pattern blocks students will find a shape or shapes that has: only right angles, only acute angles, only obtuse angles. Put shapes together to make straight angles. Find three shapes that have two acute angles and two obtuse angles.

Working in pairs, introduce the concept of complementary angles. Find two pattern block shapes that have corners that make complementary angles when matched. Use a shape with a $90^{\circ}$ angle to prove your answer. How many different combinations of shapes will make complementary angles? (3)

Introduce the concept of supplementary angles. Find two pattern block shapes that will make supplementary angles when their corners are matched up. How many combinations can you find? (9)

Introduce the concepts of vertical and corresponding angles if students are ready.

Step 4 - Triangles are closed figures made up of three angles. They can be classified by properties of angles:
-- One angle of a right triangle is equal to $90^{\circ}$.
-- All angles of an acute triangle are less than $90^{\circ}$.
-- One angle of an obtuse triangle is greater than $90^{\circ}$.
Triangles can also be classified by properties of sides:
-- Equilateral triangles have all sides congruent.
-- Isosceles triangles have two sides congruent.
-- Scalene triangles have no sides congruent.
Show or find examples of these in the classroom and everyday objects.

If students are new to this terminology and are still a bit confused by the concepts, it may be helpful to complete the Types of Triangles and Their Properties chart together.

Step 5 - Venn diagrams provide a visual organizer to help students consider relationships. The handout, Using Venn Diagrams to

Reason about Shapes, will help students formulate arguments to justify placement of various triangles in the Venn diagram.
Encouraging the students to illustrate their reasoning with diagrams can help them articulate their arguments. Consider adding a fourth area, representing everything outside A and B. A variation of this Venn diagram activity is to use types of quadrilaterals, such as rectangles, squares, rhombuses, trapezoids or kites. This variation helps students focus on both the angles and the sides of the shapes.

This activity may be challenging for some students to visualize. Using manipulatives of the actual shapes would be a way to scaffold the learning for lower level students.

Teacher Note For additional practice with angles and triangles, refer to The GED Math Problem Solver by Myrna Manly, pages 38-47.

Step 6 - Remind students of how Clara used word-of-mouth from slaves to design her quilt to freedom. Just as Clara designed a quilt, students will be able to design their own quilts.
Challenge each student to create an original design that has a special pattern or a meaning using geometric shapes and tessellations and no more than four colors or work in cooperative groups to make a quilt that describes the journey Clara might have taken.

Step 7-Have each student explain their pattern in paragraph form, making sure to include the meaning and the mathematical significance of their pattern. Provide an opportunity for the students to share their designs and the meaning they included for the pattern they created.

Assessment/Evidence (based on outcome)
Patterns created by manipulatives
Types of Triangles and Their Properties Chart
Using Venn Diagrams to Reason About Shapes Handout
Original quilt designs and paragraph explanations
Teacher observation

Teacher Reflection/Lesson Evaluation
Not yet completed.

## Next Steps

The lesson Flips, Slides and Turns on geometric transformations of reflections, translations and rotation could follow this lesson for students ready to move to the next math level. If studying the Civil War unit, the next lesson will be on the Underground Railroad. Quilting Geometry Learning Objects will give students additional practice with angular measurement.

## Technology Integration

Venn Diagram Teaching Strategy http://literacy.kent.edu/eureka/strategies/venn diagrams.pdf

## Quilt Resources

Learn How to Make a Quilt http://www.learnhowtomakequilts.com/
Underground Railroad Quilt Code Myth http://www.ugrrquilt.hartcottagequilts.com/
African-American Historical Quilts http://quiltethnic.com/historical/
Antique Geometric Quilt Designs http://earlywomenmasters.net/quilts/
Underground Railroad Quilts \& Abolitionist Fairs http://www.womenfolk.com/quilting history/abolitionist.htm
The Underground Railroad Quilt Code http://ugrrquilt.hartcottagequilts.com/betsy\ ross\ redux.pdf
Owen Sound's Black History Quilt Code http://www.osblackhistory.com/quilts.php

## Geometry Options

Polygon Quilt Game http://teams.lacoe.edu/documentation/classrooms/amy/geometry/3-4/activities/quilt.html Geometry in the Adult Education Classroom http://literacy.kent.edu/Oasis/Pubs/mathwinter01.pdf

## Literature Annotation

This story is based on a true, little-known chapter in African American history. As a seamstress in the Big House, Clara knows she's better off than the slaves who work the fields. But slavery has separated Clara from her mother, and she can never be happy without her. Clara dreams that they will be reunited one day and run away together - north to freedom. Then Clara hears two slaves talking about how they could find the Underground Railroad if only they had a map. In a flash of inspiration, she sees how to use the cloth in her scrap bag to sew a map of the land - a freedom quilt - that no master will ever suspect is a map to freedom.

## Trade Book Additional Resources

The Secret to Freedom by Marcia Vaughn

## Show Way by Jacqueline Woodson

The Log Cabin Quilt by Ellen Howard
Eight Hands Round: A Patchwork Alphabet by Ann Whitford Paul
Secret to Freedom by Marcia Vaughan
Hidden in Plain View: A Secret Story of Quilts and the Underground Railroad by Jacqueline L. Tobin

## Purposeful/Transparent

While studying the Underground Railroad, students become interested in quilts and the quilt code, giving the teacher the opportunity to extend the lesson into the academic area of mathematics.

## Contextual

Using authentic quilts as examples and allowing students to produce a quilt design gives students the chance to better understand how quilts fit into our history. Reading the piece of literature also ties the two content areas together and provides a background for their understanding.

## Building Expertise

Angles and triangles would have been previously introduced, but this lesson allows students to manipulate and visualize their relationship.

Teachers are always looking for ways to improve their teaching and to help students understand mathematics. Research in many countries supports the idea that mathematics instruction and student mathematics understanding will be more effective if manipulative materials are used. They help students move from the concrete to abstract level in their mathematical thinking. Sowell (1989) concluded that mathematics achievement is increased by the long term use of manipulatives and that student attitudes toward mathematics are improved when they use manipulatives.

Manipulative materials are concrete models that involve mathematics concepts. They appeal to several senses, and students can touch them and move them around. Each student (or pair of students) needs material to manipulate independently; teacher demonstrations are not sufficient. There are many benefits to using manipulative materials; however the most important is to foster understanding of mathematical concepts. We often forget how hard it is for students to "see" a concept. Looking at pictures does not produce the same level of understanding for our students.

Time is a critical factor in the ABLE classroom, and much manipulative work will be teacher directed. Manipulatives are especially useful when teaching geometry and measurement. Actually putting square tiles into rectangles and looking at the relationship between perimeters and areas is powerful. Trying to fit 12 square inches into a square foot often results in an "aha" moment.

Manipulatives are not just for children; adults benefit from them, too. There may be some students who resist using the materials, just as there are some students who resist group work. Allowing them to work independently is fine; it is a rare student who doesn't eventually become intrigued with what is happening and join the others.

Many books are available at each regional resource center to help teachers understand the various manipulatives and how they can be used. Additional ideas many be found by searching for a particular manipulative on-line.


Different manipulatives (tangrams, pattern blocks, square tiles) can all reinforce concepts important for the GED and life. Be on the look out for students' questions and misunderstandings that a short manipulative session can resolve. Allow students to use manipulatives when problem solving; have them readily available.

Every year, the Math Kick-Off professional development sessions include manipulatives as part of the training. Teachers receive materials to use in their classrooms, along with techniques to use them. Regional resource centers also have materials that can be borrowed; however, teachers need basic supplies in the classroom at all times. Many manipulatives can be made inexpensively and even purchased with classroom funds.

Math Manipulatives in the ABLE Classroom. The starred (*) materials should be available in every classroom every day.

## Paper *

- Areas of triangles, rectangles, and parallelograms
- 180 degrees in any triangle
- One square inch, one square foot
- Perfect triples for the Pythagorean theorem
- Magnified inch

Square tiles (preferably $1^{\prime \prime}$ ) *

- Perimeter
- Area
- Squares and square roots
- Fractions
- Percents
- Probability
- Algebraic patterns


## Graph Boards *

- Perimeter
- Area
- Fractional area
- Plotting points on the coordinate grid
- Slope
- Y-intercept
- Reflections and translations


## Pattern Blocks

- Patterns
- Fractions
- Shapes
- Angles

Cubes (preferably $1^{\prime \prime}$ )

- Volume
- Cubes and powers


## Tangrams

- Fractions
- Shapes
- Angles


## Spinners

- Probability

Dice (preferably ten-sided)

- Fractions
- Probability


## Cuisenaire Rods (very expensive)

- Fractions


## Fraction Circles

## Fraction Squares

Fraction Strips (Family Math, Jean Steinmark)

## Algebra Tiles

- Factoring
- Polynomial and monomial operations

If computers are available for student use, teachers may use a new category of visual representations called virtual manipulatives. These dynamic visuals can be manipulated in the same ways that a concrete manipulative can. Students can use a computer mouse to actually slide, flip and turn the dynamic visual representation as if it were a three-dimensional object. A virtual manipulative is an interactive, Webbased visual representation of a dynamic object that presents opportunities for exploring concepts and constructing meaning. Currently, virtual manipulatives are modeled on the concrete manipulatives commonly used in classrooms, such as pattern blocks, tangrams, fraction bars, geoboards, and geometric solids.

The best virtual manipulative sites have a variety of dynamic features that allow users to perform various mathematical investigations. One of the most extensive collections is the National Library of Virtual Manipulatives http://nlvm.usu.edu/en/nav/index.html. Users can choose from the following strands: numbers and operations, algebra, geometry, measurement, data analysis and probability.

Check out these additional sites: Dr. Super's Virtual Math Manipulatives Project
www.galaxy.gmu.edu/~drsuper; Manipula Math www.ies.co.jp/math/iava/index.html; Geometry Applet http://aleph0.clarku.edu/~djovce/iava/Geometry/Geometry.html; Algebra Tiles www.coe.tamu.edu/ ~strader/Mathematics/Algebra/AlgebraTiles/AlgebraTiles1.html; Educational Java Site, Arcytech www.arcytech.org/iava and No Matter What Shape Your Fractions Are In http://math.rice.edu/~lanius/Patterns/index.html

|  | Acute Angles | Right Angles | Obtuse <br> Angles | Two Sides <br> Congruent | Three Sides <br> Congruent |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Equilateral <br> Triangle |  |  |  |  |  |
| Isosceles |  |  |  |  |  |
| Triangle |  |  |  |  |  |
| Scalene |  |  |  |  |  |
| Triangle |  |  |  |  |  |
| Acute <br> Triangle |  |  |  |  |  |
| Right <br> Triangle |  |  |  |  |  |
| Obtuse <br> Triangle |  |  |  |  |  |

Answer using All, None, Sometimes, Yes, No, 1, 2 or 3

Types of Triangles and Their Properties Teacher Answer Key

|  | Acute Angles | Right Angles | Obtuse Angles | Two Sides <br> Congruent | Three Sides <br> Congruent |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Equilateral <br> Triangle | All | None | None | Yes | Yes |
| Isosceles <br> Triangle | 2; sometimes 3 | Sometimes 1 | Sometimes 1 | Yes | Sometimes |
| Scalene <br> Triangle <br> Acute | 2; sometimes 3 | Sometimes 1 | Sometimes 1 | No | No |
| Triangle | All | None | None | Sometimes | Sometimes |
| Right <br> Triangle | 2 | 1 | None | Sometimes | No |
| Obtuse <br> Triangle | 2 | None | 1 | Sometimes | No |

## USING VENN DIAGRAMS TO REASON ABOUT SHAPES

As you are thinking about the properties of triangles, use the Venn diagram below to label sets $A$ and $B$ with any of the following types of triangles: acute, equilateral, isosceles, obtuse, right or scalene.
Set A Set B


Explain why you labeled the Venn diagram as you did.

Draw examples of appropriate triangles in regions 1,2 and 3.
What are the relationships among these sets of triangles?
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$\qquad$

http://www.wisconline.org

## Logical Reasoning in Speeches

Author: Dr. Cynthia Ellenbecker
School: Lakeshore Technical College
Description: This activity is cognitive. Students are introduced to credibility in public speaking via primary and secondary research. For example, information brochures, although the student may locate the material himself/herself, it is still secondary research (one step away from first-hand experience).
http://www.wisc-online.com/objects/index_tj.asp?objID=SPH2001

## Searching the Internet

Author: Leanne Healy
School: Western Wisconsin Technical College
Description: In this interactive object, learners answer questions about doing research on the Internet. http://www.wisc-online.com/objects/index_tj.asp?objID=IAT204

