<table>
<tr><td rowspan="2"></td></tr>
</table>

	Student/Class Goal
# LET'S PLOT POINTS	Students often have little experience with coordinate planes and are concerned about these kinds of questions on the GED test.

Outcome (*lesson objective*)	**Time Frame**
Students will understand and be able to correctly solve problems using coordinate geometry to describe spatial relationships. They will specify locations using common language and geometric vocabulary, using coordinate systems to specific locations while finding the distance between points along horizontal and vertical lines of the coordinate system**.**	Several classes. This concept must be reinforced repeatedly over many sessions.

Standard *Use Math to Solve Problems and Communicate*	**NRS EFL 3-5**

Number Sense	Benchmarks	Geometry & Measurement	Benchmarks	Processes	Benchmarks
Words to numbers connection		Geometric figures		Word problems	
Calculation		Coordinate system	3.7, 4.8, 5.7	Problem solving strategies	
Order of operations		Perimeter/area/volume formulas		Solutions analysis	
Compare/order numbers		Graphing two-dimensional figures	3.9, 4.10, 5.9	Calculator	
Estimation		Measurement relationships		Mathematical terminology/symbols	3.23, 4.29
Exponents/radical expressions		Pythagorean theorem		Logical progression	
Algebra & Patterns	**Benchmarks**	Measurement applications		Contextual situations	
Patterns/sequences		Measurement conversions		Mathematical material	
Equations/expressions	3.15, 4.16	Rounding		Logical terms	
Linear/nonlinear representations		**Data Analysis & Probability**	**Benchmarks**	Accuracy/precision	3.26, 4.33, 5.34
Graphing	4.18	Data interpretation		Real-life applications	3.27, 4.34
Linear equations	4.19	Data displays construction		Independence/range/fluency	3.28, 4.35
Quadratic equations		Central tendency			
		Probabilities			
		Contextual probability			

Materials
Coordinate Grid Paper
Coordinate Geometry Vocabulary Handout
Odd Word Out Activity
Coordinate Grid Pictures Handout
Let's Plot Points Rubric
Analytic Geometry Teacher Resource
Battleship Grid Paper

Learner Prior Knowledge
Basic understanding of number lines that include both positive and negative values. Comparing a number line to a thermometer on its side can help students understand this concept.

Instructional Activities
Step 1 - Distribute coordinate planes to the class. Introduce or review the following vocabulary: number line, positive numbers, negative numbers, coordinate grid, x-axis, y-axis, origin, quadrants, ordered pair, coordinate, x-coordinate and y-coordinate. The handout, *Coordinate Geometry Vocabulary*, includes vocabulary and concepts students need to know in order to understand the coordinate system and can be distributed as a future reference.

To clarify the meaning of these concepts, students can compare how these vocabulary words are similar to or different from other words by playing *Odd Word Out*. Before class, create word cards using *Odd Word Out* activity sheet, making a set for pairs of

students. Model this activity first by selecting three or four words, two or three of which share a common characteristic. Choose "number line, x-axis, y-axis and coordinates." The odd word could be "coordinates" because it has to do with specific numbers and the other three deal with lines. Students might also choose "x-axis and y-axis" because they are hyphenated words. Ask students to choose 1 word from the set that does not fit and then explain why.

Students can also play "Pair 'Em Up" with the words. In this case, pairs of students find two words that are related somehow and challenge other students to guess the relationship. You can note students' understanding of the key vocabulary words and also track students' classification skills.

Step 2 - Using an overhead or a large chart paper, plot several points together as a class. To help students plot points the following tips may be helpful:

- To remember which axis is which, remind the students that the bottom of the "Y" goes up and down like the Y axis does.
- To help students remember which coordinate comes first in an ordered pair, remind them that x comes before y in the alphabet; just like it does in an ordered pair!
- Reinforce the importance of the order of the x and y axis. Graph (1,3), (2,4), and (3,5) on the coordinate grid. Connect the points with a line. Next, graph (3,1), (4,2), and (5,3) on the same grid. Connect these points with a different color. Discuss with the students what they observe.

To plot a point, first start at the origin. Look at the first coordinate in the pair. If it is a positive number, count over that many points to the right. If it is a negative number, count that many points to the left. Next look at the second number in the ordered pair. If it is positive, go up that number of points from the point you are at on the x-axis. If the number is negative, go down that many points from your location to find the location. Remember, **do not** count the point you are on when you count the points (left or right and up or down).

Step 3 – Students will need repeated opportunities to practice this concept. Try several of the following activities to help your class become fluent with plotting points on the coordinate plane.

1) If space is available, use a roll of masking tape to create a large coordinate grid on the floor. Give each student an ordered pair and let them stand at "their point."
2) Laminated grid boards, if available, are very useful to practice plotting points.
3) Distribute a paper copy of a coordinate grid. Provide the students with a set of points that will result in a picture when plotted. The *Coordinate Grid Pictures* handout contains the coordinates for 4 pictures, and the links below include sources for additional coordinate grid pictures.
4) Encourage the students to draw a simple design that extends into all four quadrants of the coordinate grid. The design should consist of a series of dots connected by straight line – "dot-to-dot" style with no curved lines. On the blank side of another piece of graph paper, have students write directions for their design, consisting of sequential ordered pairs that when graphed and connected with straight lines, will duplicate their design. Students then exchange their *directions only* with any student who has not seen their design. Following the written directions, students will graph each ordered pair and connect the points with straight lines using a ruler. Compare the completed design with the classmate who wrote the directions. If their designs are not congruent (same shape and size), have the two troubleshoot and problem solve whether the problem was in the directions, or in the way the student read and graphed the directions. Discuss the implications these scenarios could have in real-world situations if either the directions were incorrect or unclear, of if they were not followed properly (e.g. an engineer writing directions for a mechanic to build a machine the engineer designed, an architect drawing plans for a builder to follow, etc.)
5) Play the game Battleship on a coordinate grid. Use the worksheet generator to make paper for the "Battleship" game. Pick four on a page, 1/4 inch squares, 6 x 8 unit quadrants and numbered, and you will end up with a worksheet that can be cut lengthwise so two students can play. Each person will need a sheet of paper with two coordinate grids. On the top grid you will plot your ships (Aircraft carrier-5 points long, Battleship-4 points long, Submarine-3 points long, Destroyer-3 points long and PT Boat-2 points long). On the lower grid you will indicate where you have shot missiles trying to sink your partner's fleet. Be sure to mark if part of one of your ships was hit and what boats you have hit. The partners take turns calling out points (i.e. x-coordinate 5, y-coordinate -2) until one player has sunk all the parts of his partner's ships. Since players must call out the coordinates of each point they wish to guess, the game provides lots of practice using the coordinate grid. It might be a good idea to write down the points each player calls in order to mediate possible disputes later.

Play the game Equation Maker (see website) with the class. Write a simple equation on the board (i.e. 3 + x = y or 2x = y). Give the class the value of the x-coordinate, and have the students find the value of y. Let the students plot the ordered pair. Substitute several numbers into each equation. Connect the points graphed for each equation to introduce students to the relationship

between a graph and an equation.

Step 4 - Discuss situations that require knowledge of coordinate grids. Tool and Die makers use a 3-dimensional grid system to program the necessary cuts on the machines that make the dies. We use a grid system when we search for a city on a road map. Latitude and longitude lines that are used to find locations on maps work like a coordinate grid system. The I-phone technology is based on a coordinate grid system.

TEACHER NOTE See the *Analytic Geometry* Teacher Resource for additional background on coordinate planes and several applications to real-life activities that can be shared with students.

Step 5 - Assess the students in two ways. Using the *Let's Plot Points* rubric, have students draw a four-quadrant grid with all quadrants labeled, x-axis, y-axis Then plot two given points (for example, A (3,5); B (3,-4); C (0,4); etc.) in each quadrant and label them with the correct ordered pairs. Students can also complete several exercises from a GED text requiring them to plot a specific point or identify the ordered pair of a specific point. Ask students to explain how they find a point and how they identify the coordinates in an ordered pair.

TEACHER NOTE Higher level students can obtain more practice with linear functions at Linear Functions and Their Applications in Developmental Math .

Assessment/Evidence (*based on outcome*)
Students plot points to successfully complete a coordinate picture or find a point on a coordinate grid.
Teacher observes and makes anecdotal notes.

Teacher Reflection/Lesson Evaluation
After earning his GED a former student entered a program for tool and die makers. He mentioned that the skills he had learned in the GED/ABE class on plotting points on a coordinate grid were very helpful in his training program. It was the skill the class started out with. My students love doing the coordinate pictures. They have fun and feel successful as they learn to plot points on a coordinate grid.

Next Steps
Help students understand transformations (flips, turns and slides) and missing points in shapes on the coordinate plane. *Flips, Slides and Turns* could follow this lesson. Let's Plot Points Learning Objects will give students additional practice in identifying points on a coordinate plane and finding vectors on a Cartesian coordinate system.

Technology Integration
Coordinate Grid Paper http://www.donnayoung.org/math/c-grids.htm
Coordinate Grid Paper http://www.printfreegraphpaper.com/
Coordinate Grid Paper http://themathworksheetsite.com/coordinate_plane.html
Battleship http://themathworksheetsite.com/coordinate_plane.html
Dinosaur Picture http://www.mathsisfun.com/t_rex.html
Cat Picture http://www.mathcats.com/crafts/grids/catgrid2.html
Ice Cream Sundae Picture http://www.uen.org/Lessonplan/preview.cgi?LPid=15431
The Math Worksheet Site http://themathworksheetsite.com/   Online math worksheet generator – subscription needed
Linear Functions and Their Applications in Developmental Math http://www.texascollaborative.org/FordHodgeModule/index.php
Equation Maker http://www.codecogs.com/latex/eqneditor.php
Equation of a Line Maker http://dooleymath.com/Algebra/EquationMaker.html

---

Purposeful/Transparent
The activities help the students learn and practice plotting points.

Contextual
Students discuss potential uses in their lives for knowledge of coordinate grids, such as maps and navigation.

Building Expertise
Students can relate their understanding of ordered pairs to algebraic equations.

Coordinate Geometry Vocabulary

Number line - *A line that shows numbers in order.*

Positive numbers - *Numbers greater than zero.*

Negative numbers - *Numbers less than zero.*

Coordinate Plane [Rectangular Coordinate System] - *Two number lines (including both positive and negative numbers) perpendicular to one another and intersecting at the zero point of both lines.*

Coordinate Grid - *A coordinate plane placed on graph paper.*

Axes - *The names given to the number lines that run horizontally (x) and vertically (y) on the coordinate plane.*

X-axis - *The horizontal line on a coordinate plane. The positive numbers are located to the right of the origin, and the negative numbers are to the left of the origin.*

Y-axis - *The vertical line on a coordinate plane. The positive numbers are located above the origin, and the negative numbers are below the origin.*

Origin - *The point where the x-axis and the y-axis intersect on the coordinate plane. The coordinates of the origin are (0,0).*

Ordered Pair - *A pair of numbers used to locate a point on a coordinate grid, such as (5,-2). The x-axis coordinate is always first because "x" comes before "y" alphabetically.*

Coordinates - *One of the numbers in an ordered pair. The x value is the first coordinate of the pair and the y value is the second coordinate.*

X-coordinate - *Identifies the position of the point along the horizontal (x) axis.*

Y-coordinate - *Identifies the position of the point along the vertical (y) axis.*

Quadrants - *The four regions of the coordinate plane that the axes divide it into. There are 4 quadrants, labeled in counter-clockwise order with quadrant I in the upper right corner. I (+, +), II (-, +), III (-,-), IV (+,-). The sign of the x-value and y-value are noted in the ordered pair (x,y).*

Coordinate Geometry Vocabulary Handout

Number Line	**X-axis**
Positive Numbers	**Y-axis**
Negative Numbers	**Ordered Pair**
Coordinate Plane	**Coordinates**
Coordinate Grid	**X-coordinate**
Axes	**Y-coordinate**
Origin	**Quadrants**

Odd Word Out Activity

Coordinate Grid Pictures

Directions: The following lists of coordinates each create a picture. Be sure to join each point to the previous point. When a point says start next to it, do not join this point to the previous point as you will be starting a new part of the picture.

Picture 1	Picture 2	Picture 3	Picture 4	Picture 4 (continued)
Start:	Start:	Start:	Start:	Start:
(-6,-4)	(-1, 7)	(-3, 1)	(-1, 6)	(3, -3)
(7,-4)	(-4, 6)	(-4, 2)	(1, 6)	(5, -5)
(6,-7)	(-6, 4)	(-5, 4)	(3, 5)	6, -6)
(-5,-7)	(-7, 2)	(-5, 6)	(4, 3)	(7, -6)
	(-7, -1)	(-4, 8)	(5, 1)	(7, -5)
Start:	(-6, -4)	(-2, 9)	(6, -2)	(5, -3)
(3, 7)	(-4, -6)	(2, 9)	(6, -3)	
(3,-4)	(-1, -7)	(4, 8)	(-7, 3)	Start:
	(-3, -5)	(5, 6)	(-7, 2)	(5, 1)
Start:	(-4, -2)	(5, 4)	(-6, 0)	(6, 2)
(3, 7)	(-4, 2)	(4, 2)	(-5, 3)	(7, 3)
(1, 5)	(-3, 5)	(3, 1)	(-3, 5)	(8, 3)
(-2, 1)	(-1, 7)	(-3, 1)	(-1, 6)	(9, 2)
(-4,-2)				(8, 1)
	Start:	Start:	Start:	(7, 1)
	(4, 6)	(-3, 1)	(-7, -3)	(6, -2)
	(5, 2)	(-2, -3)	(-8, -3)	
	(8, 2)	(0, -9)	(-9, -1)	Start:
	(7, -1)	(2, -3)	(-7, -2)	(8, 2)
	(8, -5)	(3, 1)		
	(4, -3)		Start:	
	(0, -5)		(-6, -3)	
	(1, -1)		(-8, -5)	
	(-1, 2)		(-8, -6)	
	(2, 2)		(-7, -6)	
	(4, 6)		(-6, -5)	
			(-4, -3)	

Let's Plot Points Rubric

Evaluate the students' understanding of coordinate planes by using the following rubric.

4 Complete Understanding and Application	Student accurately plots points and reads the coordinates on a coordinate grid.
3 Substantial Understanding and Application	Student plot points and reads the coordinates of points on a coordinate plane, but is not always accurate.
2 Some Understanding and Application	Student can occasionally (25%) plot points and read coordinates on a coordinate grid correctly.
1 Little Understanding or Application	Student does not plot points or read coordinates of points correctly.

Student's Evaluation _____ **Teacher's Evaluation** _____

Comments and Observations

Analytic Geometry

Cogito ergo sum. "I think, therefore I am." *Descartes,* 1637.

Background for Teachers

Analytic geometry, otherwise known as coordinate geometry or Cartesian geometry, the brainchild of Pierre de Fermat and Rene Descartes in 1637, is the study of points, curves and lines defined by algebraic expressions. They cleverly married algebra and geometry with their development of the Cartesian plane. This investigation of geometric objects springs from the idea that any point in two-dimensional space can be represented by two numbers and any point in three-dimensional space by three. Because lines, circles, spheres and other figures can be thought of as collections of points in space that satisfy certain equations, they can be explored via equations and formulas rather than graphs. Analytic geometry can be explained more simply: it is concerned with defining geometrical shapes in a numerical way and extracting numerical information from that representation. The numerical output, however, might also be a vector or a shape. Some consider that the introduction of analytic geometry was the beginning of modern mathematics.

Themes of Analytical Geometry

You can use a *coordinate* grid to locate points on the plane. The x-axis and the y-axis are number lines. They intersect at right angles at their zero points, the *origin*. Any point can be located using an ordered pair. The first coordinate tells you how far to move on the x-axis from the origin. Coordinates of points to the right of the origin are positive numbers. Coordinates of points to the left are negative numbers. The second coordinate tells you how far to move on the y-axis from the origin. Coordinates of points up from the origin are positive numbers. Coordinates of points down from the origin have coordinates that are negative numbers. Measures of distance can be found between two points.

Concepts to also be studied include: lines (vertical, horizontal, parallel, perpendicular, intersecting) and relationships, rays, line segments, slope and intercept of a line (equation of slope-intercept form) and angles.

Many of these problems involve linear algebra. **Linear algebra** is the branch of mathematics concerned with the study of vectors, vector spaces (also called *linear spaces*), linear maps (also called *linear transformations*), and systems of linear equations. Vector spaces are a central theme in modern mathematics; thus, linear algebra is widely used in both abstract algebra and functional analysis. Linear algebra also has a concrete representation in analytic geometry and it is generalized in operator theory. It has extensive applications in the natural sciences and the social sciences, since nonlinear models can often be approximated by linear ones.

Applications of Analytic Geometry

Mathematical applications of analytic geometry lie mainly in relating algebra and geometry, but the concepts are also useful for solving problems such as calculus on the Cartesian plane.
Analytic geometry can also be used in the practical world. One very important example is in cartography. It can be used in the location of places and points on a topographical map. The concept of longitude and latitude is also based on analytic geometry; which is also used in navigation.

Archeologists use Cartesian coordinate systems to identify the location of artifacts. If you worked in a manufacturing plant, you might have to consider this problem:

> Robots move parts along an assembly line and their movements are described in a coordinate plane. The range of Robot A is from (7,8) to (12,8); Robot B is from (5,3) to (5,-2); and Robot C is from (7,-7) to (12,7). Using a Cartesian coordinate grid, what is the range for each robot?

Analytic Geometry Teacher Resource

Plotting Points on a Coordinate System
Author: Francine Nettesheim
School: Northcentral Technical College
Description: Students identify points on a coordinate system.
http://www.wisc-online.com/objects/index_tj.asp?objID=ABM201

Vectors in Polar Form
Author: Jolene Hartwick
School: Western Wisconsin Technical College
Description: The learner places given vectors in correct positions on the Cartesian coordinate system. The vectors contain both positive and negative magnitude and direction.
http://www.wisc-online.com/objects/index_tj.asp?objID=TMH2602

Let's Plot Points Learning Objects