How Does That Work?

Student/Class Goal
Students will be able to answer the question, *how does that work?* when they get an e-mail with a “think of a number” problem.

Outcome (*lesson objective*)
Students will be able to translate a word situation into an algebraic expression and then simplify it.

Time Frame
One class (1 hour)

Standard *Use Math to Solve Problems and Communicate*

NRS EFL 4-6

Number Sense	Benchmarks	Geometry & Measurement	Benchmarks	Processes	Benchmarks
Words to numbers connection		Geometric figures		Word problems	4.25, 5.25, 6.26
Calculation	4.2, 5.1, 6.1	Coordinate system		Problem solving strategies	
Order of operations		Perimeter/area/volume formulas		Solutions analysis	4.27, 5.27, 6.28
Compare/order numbers		Graphing two-dimensional figures		Calculator	4.28, 5.28, 6.29
Estimation		Measurement relationships		Mathematical terminology/symbols	4.29, 5.29, 6.30
Exponents/radical expressions		Pythagorean theorem		Logical progression	4.30, 5.30, 6.31
Algebra & Patterns	**Benchmarks**	Measurement applications		Contextual situations	4.31, 5.31, 6.32
Patterns/sequences	4.15, 5.15, 6.15	Measurement conversions		Mathematical material	
Equations/expressions	4.16, 5.16, 6.16	Rounding		Logical terms	
Linear/nonlinear relationships	4.17, 5.17, 6.17	**Data Analysis & Probability**	**Benchmarks**	Accuracy/precision	
Graphing		Data interpretation		Real-life applications	4.34, 5.35, 6.36
Linear equations		Data displays construction		Independence/range/fluency	4.35, 5.36, 6.37
Quadratic equations		Central tendency			
		Probabilities			
		Contextual probability			

Materials
Chalk board, white board or flip chart
Think of a Number Email Handouts
Think of a Number *2 Handout
Think of a Number *3 Handout
Think of a Number *4 Handout
Think of a Number *5 Handout
Calendar Algebra Handout
Your Age by Eating Out Email Handout
Your Age by Chocolate Math Handout
Teaching Algebraic Thinking Skills Teacher Resource
How Does That Work Learning Objects

Learner Prior Knowledge
Basic understanding of variables – that a number can be represented by a letter.

Instructional Activities
Step 1 - Discuss with students their experiences with e-mails. Have they received an e-mail where they say they can tell you your age by how many times you would like to eat out in a week or how many times you’d like to eat chocolate? The e-mail will ask you to answer questions and complete a number of operations as you scroll down the screen. Explain to the students that these “think

of a number" type problems are based on mathematics

Step 2 - Select one of the modeling examples from the *Think of a Number* handout, based on the level of your class. Teacher will read the email problem aloud as students attempt to figure out the number. When all the students have completed the computation individually or in pairs and come up with an answer to the problem, write the answers on the board. How do the answers compare? Are they the same (most will be) or similar?

Step 3 – Tell students that they can understand answers mathematically by generalizing the operations. The teacher will use a Think-Aloud Teaching Strategy <http://literacy.kent.edu/eureka/strategies/think_aloud.pdf> to demonstrate how she arrived at the answer. Use a variable for the number you pick then complete the various operations on your variable x. Be sure you discuss how each step could be expressed and then simplified along the way. Model the additional example, if desired, from the handout and then distribute the process for students to have as a reference. The discussion with the students is the most important part of this lesson. Go slowly! Be sure to discuss how the algebra proves the answer.

TEACHER NOTE If you would like more information about *Algebraic Thinking*, please refer to the Teacher Resource provided. Be sure the students understand that algebraic expressions are simply ways to express variables and symbols such as a sentence uses words.

Step 4 - Handouts 2-5 each contain a think of a number problem. Let the students, either alone or with a partner, select problems to solve. They should use several different numbers to prove that the answer will always be the same. Finally write the problem algebraically using a variable and expressions.
Answers:
Think of a Number * 2 handout - Your answer will be 1. Explain why.
Think of a Number * 3 handout - Will you always get your original number? Explain why.
Think of a Number * 4 handout - Your answer will be 18. Explain why.
Think of a Number * 5 handout - Your answer will always be 5. Explain why.

Step 5 - Calendar Algebra is a slightly different and more difficult problem and should be used as a class activity. Let students do the calculations on a 3x3 array and then solve the equation together as a think-aloud.
Calendar Algebra answer - Can you show that no matter what the 3x3 array of numbers, the answer will always be 9?

Step 6 - Select *Your Age by Eating Out* email handout for the students to complete without assistance. Ask that each student write a few sentences explaining why the answer to the problem can always be proven using algebra.
For example, in this handout the math could be rewritten as follows when the number selected is *x*.
Step 1: $x + 4$
Step 2: $3(x + 4) = 3x + 12$
Step 3: $3x + 12 – 9 = 3x +3$
Step 4: $2(3x + 3) = 6x + 6$
Step 5: $(6x + 6) / 6 = x + 1$
Step 6: $x + 1 – x = 1$

Assessment/Evidence (*based on outcome)*
Examine the algebraic expressions the students write in step 6 and the written work explaining their reasoning.

Teacher Reflection/Lesson Evaluation
Not yet completed.

Next Steps
Search the web for more "think of a number" problems. Encourage students to write their own "think of a number" problems. The Learning Conductor offers many valuable lessons on Patterns, Functions and Algebra. You must register to use the site, although it is free for anyone in Ohio. How Does That Work Learning Objects will give students additional practice with algebraic expressions.

Technology Integration
Think of a number http://thinkofanumber.net
Think-aloud Teaching Strategy http://literacy.kent.edu/eureka/strategies/think_aloud.pdf
The Learning Conductor http://www.ohiorc.org/for/math/learningconductor/lessons.aspx

Purposeful/Transparent
The activities all look at the math of how “think of a number” problems work.

Contextual
The lesson relates the study of algebra to a situation many students have faced: how a “pick a number” problem found in an e-mail works.

Building Expertise
The level of the worksheets varies, so students have an opportunity to increase their skill in rewriting word problems algebraically.

Teaching Algebraic Thinking Skills
Teacher Resource

Have you ever heard your students say, “Why do I have to learn algebra?” This is a great question that deserves an answer. Algebra is one of the cornerstones of today’s technology boom. Without algebra we wouldn’t have TV, radio, telephone, microwave ovens or other gadgets that make our modern world so comfortable and interesting.

Algebra is fundamental to understanding mathematical thought. It encompasses an understanding of patterns and functions, ways of representing and analyzing mathematical structures and situations, models that represent quantitative ideas and relationship and approaches that analyze change in a variety of situations. Most people recognize that algebra is needed by scientists or engineers, but algebraic thinking and reasoning are also used by many professions, including health care providers, graphic designers and home builders.

Algebraic thinking or algebraic reasoning involves forming *generalizations* from experiences with *number and computation*, formalizing these ideas with the use of a meaningful *symbol system*, and exploring the concepts of *pattern and function*.

But this is not the algebra you most likely experienced in high school. The focus now is on the type of thinking and reasoning that prepares students to think mathematically across all areas of mathematics. Being expected to teach algebra – at any level – can arouse anxiety. However, research support that algebraic thinking is necessary for today’s workforce. Blanton and Kaput (2003) state that teachers must find ways to support algebraic thinking and create a classroom culture that values “students modeling, exploring, arguing, predicting, conjecturing, and testing their ideas, as well as practicing computational skills.” Think about how to ‘algebrafy’ current curriculum by transforming existing activities and word problems from a single numerical answer to opportunities to discover patterns and make generalizations about mathematical facts and relationships. Encourage students to discuss why they believe a statement or solution to a problem is correct. Use the following prompts as ways to extend student thinking:

Tell me what you were thinking.
Could you solve this in a different way?
How do you know that is true?
Does that always work?

This type of relational thinking is crucial for students in the GED classroom, as well as for adults in the workplace. Research supports that without instruction, students bypass problem representation and try to just solve the problem. But before beginning the process of teaching algebra, teachers should be sure that students understand the basics. The key prerequisites for students to be successful in the study of algebra are to first understand the technical language of algebra; concept of variables; and concept of relations and functions.

When teaching algebra, teachers need to use practical experiences that go beyond the mere computation required by equations. When developing practice activities in the algebra classroom, be sure to:

- Develop processes/procedures for students to use when approaching algebraic tasks
- Create authentic exercises that highlight the critical attributes related to the concept being taught
- Provide opportunities for students to verbalize the task and predict what type of answer is expected
- Offer opportunities for students to discuss and write responses to questions dealing with key concepts being learned
- Select authentic exercises that anticipate future skills to be learned
- Design authentic exercises that integrate a number of ideas to reinforce prior learning as well as current and future concepts

Here are a couple suggestions to start to get your students involved with algebraic thinking. Be sure to set up T-charts to help organize your results. Seeing the data in chart form can help students recognize the pattern that is occurring. Be sure to help them generalize their pattern as an equation

The Handshake Problem
How many handshakes will there be if each person in your group shakes the hand of every person once? Start with a group of three people, then increase the group size by one person each time. Students can share and discuss different solution strategies. Generalize the total number of handshakes for an arbitrary sized group.

The Table Problem
How many people can be seated at a table constructed on 2 square card tables pushed together on one side. Only one person can sit on a side. And no one straddles the legs of the table. Now push 3 card tables together to make one

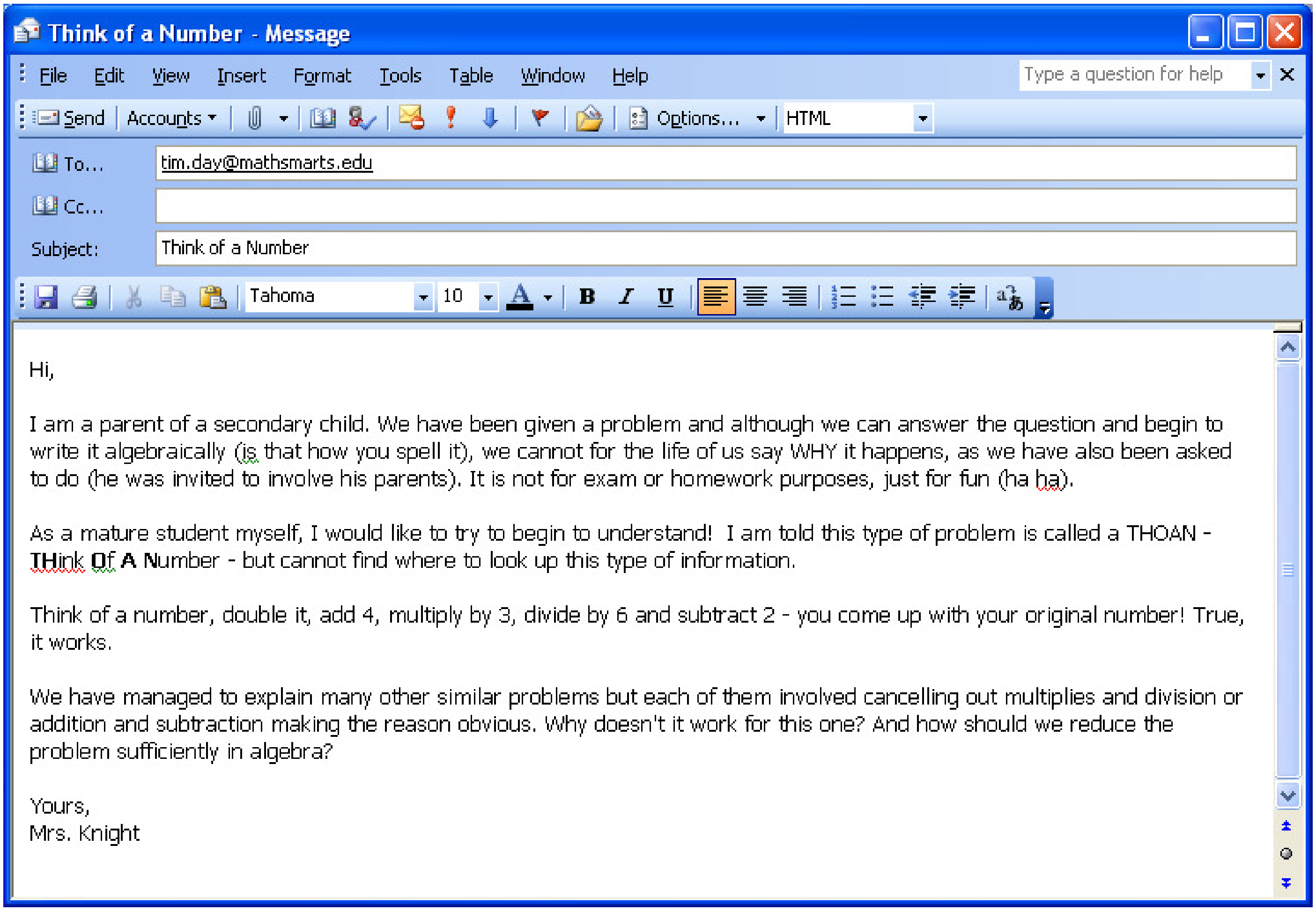
long table. How many people can be seated at this table. How many people could sit at a long table constructed of 10 card tables? How many could sit at a table made with “n” card tables?

A Visit to the County Fair

How much will it cost to ride the amusement rides at the county fair? It costs $5 to enter the fairgrounds. Amusement rides are $1.25 per ride. What is the total cost if a visitor rides 1 ride? 5 rides? 10 rides? What cost changes in the problem? What cost stays the same in the problem? If you want to spend no more than $20, how many rides can you ride? Are there other costs that influence your cost for a day at the fair?

Blanton, M. L. & Kaput, J. J. (October 2003). Developing Elementary Teachers’ “Algebra Eyes and Ears”. *Teaching Children Mathematics*, (10)2, 70-77.

Think of a Number Email Handout



Think of a Number - Message

To... tim.day@mathsmarts.edu

Cc...

Subject: Think of a Number

Hi,

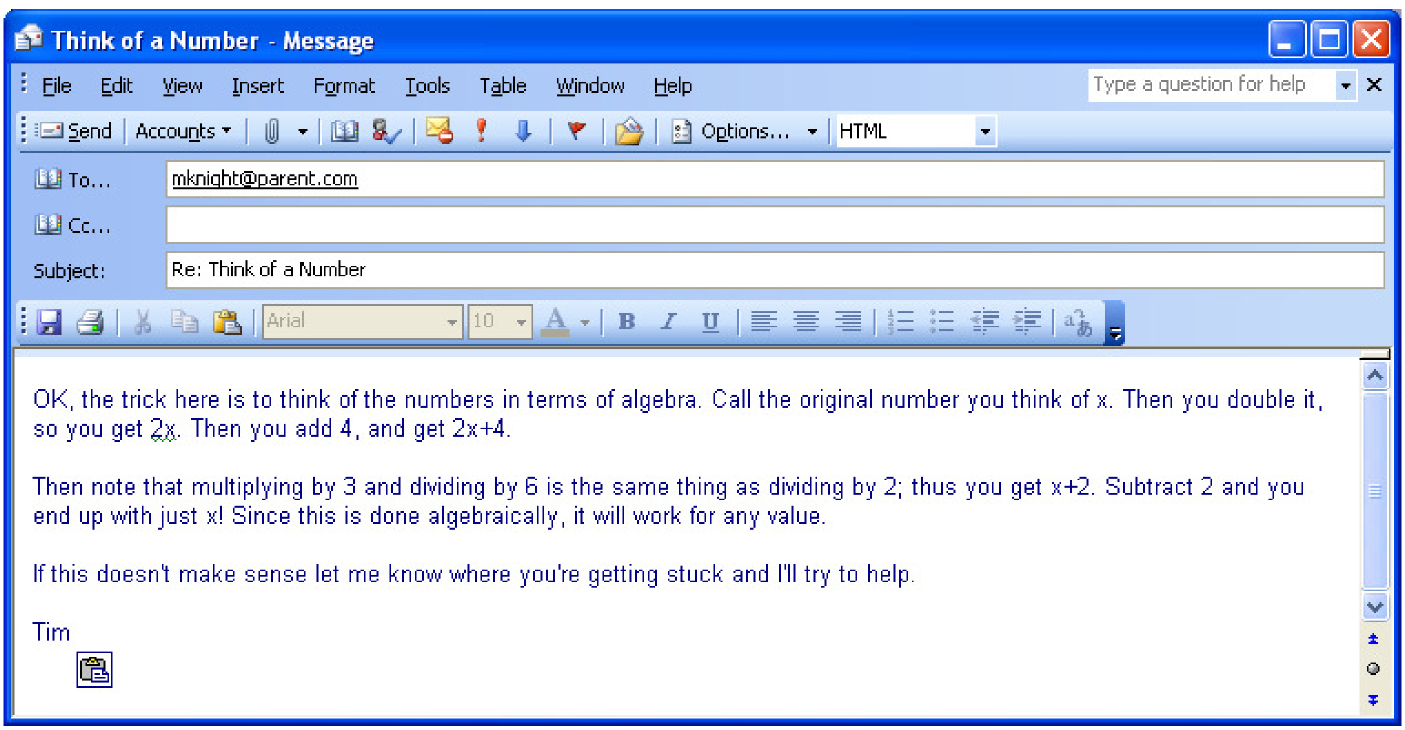
I am a parent of a secondary child. We have been given a problem and although we can answer the question and begin to write it algebraically (is that how you spell it), we cannot for the life of us say WHY it happens, as we have also been asked to do (he was invited to involve his parents). It is not for exam or homework purposes, just for fun (ha ha).

As a mature student myself, I would like to try to begin to understand! I am told this type of problem is called a THOAN - **TH**ink **O**f **A N**umber - but cannot find where to look up this type of information.

Think of a number, double it, add 4, multiply by 3, divide by 6 and subtract 2 - you come up with your original number! True, it works.

We have managed to explain many other similar problems but each of them involved cancelling out multiplies and division or addition and subtraction making the reason obvious. Why doesn't it work for this one? And how should we reduce the problem sufficiently in algebra?

Yours,
Mrs. Knight



Think of a Number - Message

To... mknight@parent.com

Cc...

Subject: Re: Think of a Number

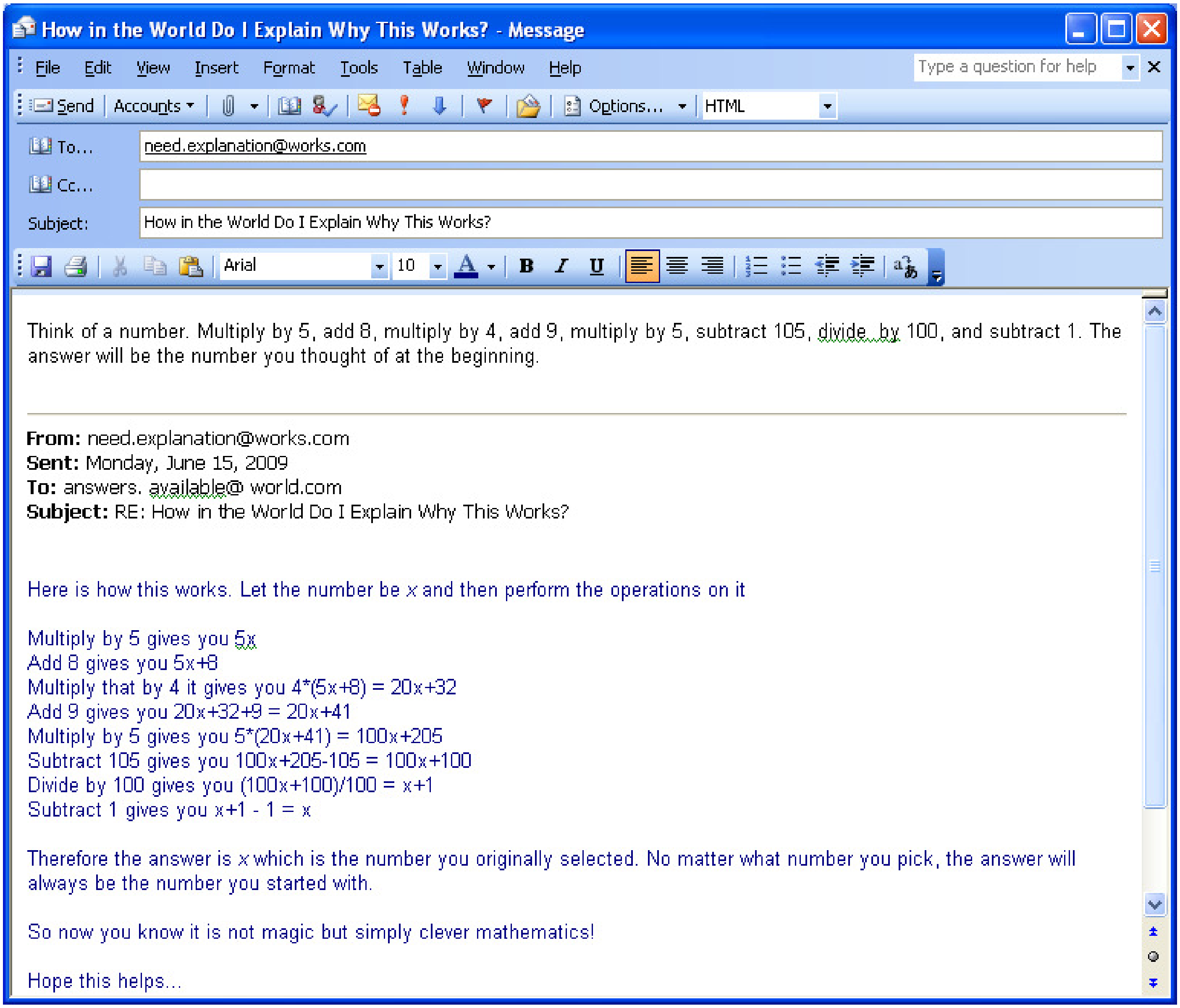
OK, the trick here is to think of the numbers in terms of algebra. Call the original number you think of x. Then you double it, so you get 2x. Then you add 4, and get 2x+4.

Then note that multiplying by 3 and dividing by 6 is the same thing as dividing by 2; thus you get x+2. Subtract 2 and you end up with just x! Since this is done algebraically, it will work for any value.

If this doesn't make sense let me know where you're getting stuck and I'll try to help.

Tim

Think of a Number Email Handout



How in the World Do I Explain Why This Works? - Message

To... need.explanation@works.com

Cc...

Subject: How in the World Do I Explain Why This Works?

Think of a number. Multiply by 5, add 8, multiply by 4, add 9, multiply by 5, subtract 105, divide by 100, and subtract 1. The answer will be the number you thought of at the beginning.

From: need.explanation@works.com
Sent: Monday, June 15, 2009
To: answers. available@ world.com
Subject: RE: How in the World Do I Explain Why This Works?

Here is how this works. Let the number be *x* and then perform the operations on it

Multiply by 5 gives you 5x
Add 8 gives you 5x+8
Multiply that by 4 it gives you 4*(5x+8) = 20x+32
Add 9 gives you 20x+32+9 = 20x+41
Multiply by 5 gives you 5*(20x+41) = 100x+205
Subtract 105 gives you 100x+205-105 = 100x+100
Divide by 100 gives you (100x+100)/100 = x+1
Subtract 1 gives you x+1 - 1 = x

Therefore the answer is *x* which is the number you originally selected. No matter what number you pick, the answer will always be the number you started with.

So now you know it is not magic but simply clever mathematics!

Hope this helps...

Think of a Number * 2

Think of a number.

Add 4.

Multiply by 3.

Subtract 9.

Multiply by 2.

Divide by 6.

Subtract the original number.

What answer did you get?

Think of a Number * 3

Think of a number.

Add 1.

Multiply by 9.

Add the original number.

Subtract 4.

Delete the ones digit.

What number do you obtain?

Think of a Number * 4

Think of a number.

Add 7 to it.

Subtract 2.

Subtract your original number.

Multiply by 4.

Subtract 2.

What is your number?

Think of a Number * 5

Think of a number between 1 and 25.

Double the number.

Add 10.

Divide by 2.

Subtract the original number.

What is the number?

Calendar Algebra

Use a calendar.

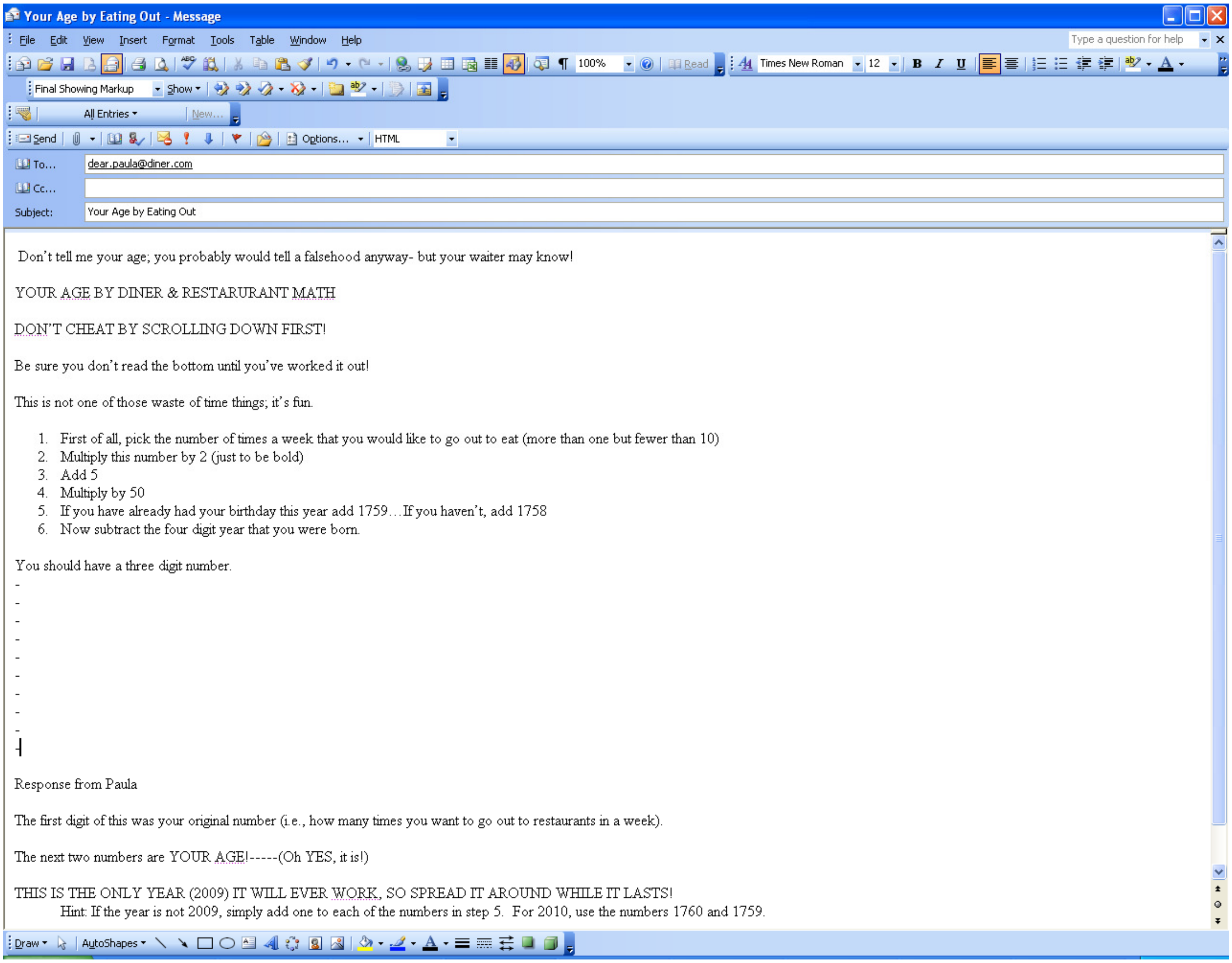
Select a 3x3 array of number from a calendar (any month, any year).

Add up the nine numbers (dates) in the array.

Divide the sum by the central number.

What number do you obtain?

Your Age by Eating Out



Your Age by Eating Out - Message

To... dear.paula@diner.com

Cc...

Subject: Your Age by Eating Out

Don't tell me your age; you probably would tell a falsehood anyway- but your waiter may know!

YOUR AGE BY DINER & RESTARURANT MATH

DON'T CHEAT BY SCROLLING DOWN FIRST!

Be sure you don't read the bottom until you've worked it out!

This is not one of those waste of time things; it's fun.

1. First of all, pick the number of times a week that you would like to go out to eat (more than one but fewer than 10)
2. Multiply this number by 2 (just to be bold)
3. Add 5
4. Multiply by 50
5. If you have already had your birthday this year add 1759…If you haven't, add 1758
6. Now subtract the four digit year that you were born.

You should have a three digit number.

-
-
-
-
-
-
-
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Response from Paula

The first digit of this was your original number (i.e., how many times you want to go out to restaurants in a week).

The next two numbers are YOUR AGE!-----(Oh YES, it is!)

THIS IS THE ONLY YEAR (2009) IT WILL EVER WORK, SO SPREAD IT AROUND WHILE IT LASTS!

Hint: If the year is not 2009, simply add one to each of the numbers in step 5. For 2010, use the numbers 1760 and 1759.

Your Age by Eating Out Handout

YOUR AGE BY CHOCOLATE MATH

Don't tell me your age; you'd probably lie anyway
-- but the Hershey Man will know!

DON'T CHEAT BY SCROLLING DOWN FIRST!

It takes less than a minute. Work this out as you read.

Be sure you don't read the next page until you've worked it out! This is not one of those waste of time things, it's fun.

1. **First of all, pick the number of times a week that you would like to have chocolate (more than once but less than 10).**
2. **Multiply this number by 2 (just to be bold).**
3. **Add 5.**
4. **Multiply it by 50 -- I'll wait while you get the calculator.**

5. **If you have already had your birthday this year add 1761.**
   If you haven't, add 1760.
6. **Now subtract the four digit year that you were born.**

You should have a three digit number.

The first digit of this was your original number
(i.e., how many times you want to have chocolate each week).

The next two numbers are

YOUR AGE! (Oh YES, it is!!!!!)

THIS IS THE ONLY YEAR (2011) IT WILL EVER WORK, SO SPREAD IT AROUND WHILE IT LASTS.

http://www.wisconline.org

Simplifying Algebraic Expressions: Addition & Subtraction
Author: Douglas Jensen & Allen Reed
School: Northeast Wisconsin Technical College
Description: Learners read definitions of the terminology associated with algebraic operations and then follow steps to simplify algebraic expressions.
http://www.wisc-online.com/objects/index_tj.asp?objID=GEM1804

Simplifying Algebraic Expressions: Division
Author: Douglas Jensen & Allen Reed
School: Northeast Wisconsin Technical College
Description: Learners read definitions of the terminology associated with algebraic operations and then follow steps to use the fundamental laws of division to simplify algebraic expressions.
http://www.wisc-online.com/objects/index_tj.asp?objID=GEM2104

Simplifying Algebraic Expressions: Multiplication
Author: Douglas Jensen & Allen Reed
School: Northeast Wisconsin Technical College
Description: Learners read definitions of the terminology associated with algebraic operations and then follow steps to use the fundamental laws of multiplication to simplify algebraic expressions.
http://www.wisc-online.com/objects/index_tj.asp?objID=GEM1904

An Introductory Algebraic Word Problem
Author: Kevin Ritzman
School: Fox Valley Technical College
Description: In this animated object, learners follow the steps for solving word problems using algebra. An introductory example is explained.
http://www.wisc-online.com/objects/index_tj.asp?objID=TMH5206