

HOW DOES THAT WORK?		Student/Class Goal Students will be able to answer the question, <i>how does that work?</i> when they get an e-mail with a “think of a number” problem.
Outcome (<i>lesson objective</i>) Students will be able to translate a word situation into an algebraic expression and then simplify it.		Time Frame One class (1 hour)
Standard <i>Use Math to Solve Problems and Communicate</i>		NRS EFL 4-6
COPs Understand, interpret, and work with pictures, numbers, and symbolic information.	Activity Addresses Components of Performance Students practice computation when working the steps of the problem.	
Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension.	Students make predictions on why a string of operations yields a particular answer. Teacher will model the problem-solving steps by using a think-aloud strategy, demonstrating explicit instructional techniques.	
Define and select data to be used in solving the problem.	Students select the appropriate number or array of numbers based on the criteria from the problem.	
Determine the degree of precision required by the situation.	Students recognize the necessity of performing the math operations correctly in order to get the “right” answer.	
Solve problem using appropriate quantitative procedures and verify that the results are reasonable.	Students select a meaningful number and perform the four basic operations to arrive at a particular answer.	
Communicate results using a variety of mathematical representations, including graphs, charts, tables, and algebraic models.	Students translate the math operations into an equation and explain why the answer comes out as it does.	
Materials Chalk board, white board or flip chart Think of a Number Email Handouts Think of a Number *2 Handout Think of a Number *3 Handout Think of a Number *4 Handout Think of a Number *5 Handout Calendar Algebra Handout Your Age by Eating Out Email Handout Your Age by Chocolate Math Handout <i>Teaching Algebraic Thinking Skills</i> Teacher Resource How Does That Work Learning Objects		
Learner Prior Knowledge Basic understanding of variables – that a number can be represented by a letter.		
Instructional Activities Step 1 - Discuss with students their experiences with e-mails. Have they received an e-mail where they say they can tell you your age by how many times you would like to eat out in a week or how many times you’d like to eat chocolate? The e-mail will ask you to answer questions and complete a number of operations as you scroll down the screen. Explain to the students that these “think of a number” type problems are based on mathematics Step 2 - Select one of the modeling examples from the <i>Think of a Number</i> handout, based on the level of your class. Teacher will read the email problem aloud as students attempt to figure out the number. When all the students have completed the computation individually or in pairs and come up with an answer to the problem, write the answers on the board. How do the answers compare? Are they the same (most will be) or similar? Step 3 – Tell students that they can understand answers mathematically by generalizing the operations. The teacher will use a Think-Aloud Teaching Strategy < http://literacy.kent.edu/eureka/strategies/think_aloud.pdf > to demonstrate how she arrived at the		

answer. Use a variable for the number you pick then complete the various operations on your variable x . Be sure you discuss how each step could be expressed and then simplified along the way. Model the additional example, if desired, from the handout and then distribute the process for students to have as a reference. The discussion with the students is the most important part of this lesson. Go slowly! Be sure to discuss how the algebra proves the answer.

TEACHER NOTE If you would like more information about *Algebraic Thinking*, please refer to the Teacher Resource provided. Be sure the students understand that algebraic expressions are simply ways to express variables and symbols such as a sentence uses words.

Step 4 - Handouts 2-5 each contain a think of a number problem. Let the students, either alone or with a partner, select problems to solve. They should use several different numbers to prove that the answer will always be the same. Finally write the problem algebraically using a variable and expressions.

Answers:

Think of a Number * 2 handout - Your answer will be 1. Explain why.

Think of a Number * 3 handout - Will you always get your original number? Explain why.

Think of a Number * 4 handout - Your answer will be 18. Explain why.

Think of a Number * 5 handout - Your answer will always be 5. Explain why.

Step 5 - Calendar Algebra is a slightly different and more difficult problem and should be used as a class activity. Let students do the calculations on a 3×3 array and then solve the equation together as a think-aloud.

Calendar Algebra answer - Can you show that no matter what the 3×3 array of numbers, the answer will always be 9?

Step 6 - Select *Your Age by Eating Out* email handout for the students to complete without assistance. Ask that each student write a few sentences explaining why the answer to the problem can always be proven using algebra.

For example, in this handout the math could be rewritten as follows when the number selected is x .

Step 1: $x + 4$

Step 2: $3(x + 4) = 3x + 12$

Step 3: $3x + 12 - 9 = 3x + 3$

Step 4: $2(3x + 3) = 6x + 6$

Step 5: $(6x + 6) / 6 = x + 1$

Step 6: $x + 1 - x = 1$

Assessment/Evidence *(based on outcome)*

Examine the algebraic expressions the students write in step 6 and the written work explaining their reasoning.

Teacher Reflection/Lesson Evaluation

Not yet completed.

Next Steps

Search the web for more "think of a number" problems. Encourage students to write their own "think of a number" problems. The Learning Conductor offers many valuable lessons on Patterns, Functions and Algebra. You must register to use the site, although it is free for anyone in Ohio. How Does That Work Learning Objects will give students additional practice with algebraic expressions.

Technology Integration

Think of a number <http://thinkofanumber.net>

Think-aloud Teaching Strategy http://literacy.kent.edu/eureka/strategies/think_aloud.pdf

The Learning Conductor <http://www.ohiorc.org/for/math/learningconductor/lessons.aspx>

Purposeful/Transparent

The activities all look at the math of how "think of a number" problems work.

Contextual

The lesson relates the study of algebra to a situation many students have faced: how a "pick a number" problem found in an e-mail works.

Building Expertise

The level of the worksheets varies, so students have an opportunity to increase their skill in rewriting word problems algebraically.

Teaching Algebraic Thinking Skills

Teacher Resource

Have you ever heard your students say, “Why do I have to learn algebra?” This is a great question that deserves an answer. Algebra is one of the cornerstones of today’s technology boom. Without algebra we wouldn’t have TV, radio, telephone, microwave ovens or other gadgets that make our modern world so comfortable and interesting.

Algebra is fundamental to understanding mathematical thought. It encompasses an understanding of patterns and functions, ways of representing and analyzing mathematical structures and situations, models that represent quantitative ideas and relationship and approaches that analyze change in a variety of situations. Most people recognize that algebra is needed by scientists or engineers, but algebraic thinking and reasoning are also used by many professions, including health care providers, graphic designers and home builders.

Algebraic thinking or algebraic reasoning involves forming *generalizations* from experiences with *number and computation*, formalizing these ideas with the use of a meaningful *symbol system*, and exploring the concepts of *pattern and function*.

But this is not the algebra you most likely experienced in high school. The focus now is on the type of thinking and reasoning that prepares students to think mathematically across all areas of mathematics. Being expected to teach algebra – at any level – can arouse anxiety. However, research support that algebraic thinking is necessary for today’s workforce. Blanton and Kaput (2003) state that teachers must find ways to support algebraic thinking and create a classroom culture that values “students modeling, exploring, arguing, predicting, conjecturing, and testing their ideas, as well as practicing computational skills.” Think about how to ‘algebrafy’ current curriculum by transforming existing activities and word problems from a single numerical answer to opportunities to discover patterns and make generalizations about mathematical facts and relationships. Encourage students to discuss why they believe a statement or solution to a problem is correct. Use the following prompts as ways to extend student thinking:

Tell me what you were thinking.

Could you solve this in a different way?

How do you know that is true?

Does that always work?

This type of relational thinking is crucial for students in the GED classroom, as well as for adults in the workplace. Research supports that without instruction, students bypass problem representation and try to just solve the problem. But before beginning the process of teaching algebra, teachers should be sure that students understand the basics. The key prerequisites for students to be successful in the study of algebra are to first understand the technical language of algebra; concept of variables; and concept of relations and functions.

When teaching algebra, teachers need to use practical experiences that go beyond the mere computation required by equations. When developing practice activities in the algebra classroom, be sure to:

- Develop processes/procedures for students to use when approaching algebraic tasks
- Create authentic exercises that highlight the critical attributes related to the concept being taught
- Provide opportunities for students to verbalize the task and predict what type of answer is expected
- Offer opportunities for students to discuss and write responses to questions dealing with key concepts being learned
- Select authentic exercises that anticipate future skills to be learned
- Design authentic exercises that integrate a number of ideas to reinforce prior learning as well as current and future concepts

Here are a couple suggestions to start to get your students involved with algebraic thinking. Be sure to set up T-charts to help organize your results. Seeing the data in chart form can help students recognize the pattern that is occurring. Be sure to help them generalize their pattern as an equation

The Handshake Problem

How many handshakes will there be if each person in your group shakes the hand of every person once? Start with a group of three people, then increase the group size by one person each time. Students can share and discuss different solution strategies. Generalize the total number of handshakes for an arbitrary sized group.

The Table Problem

How many people can be seated at a table constructed on 2 square card tables pushed together on one side. Only one person can sit on a side. And no one straddles the legs of the table. Now push 3 card tables together to make one

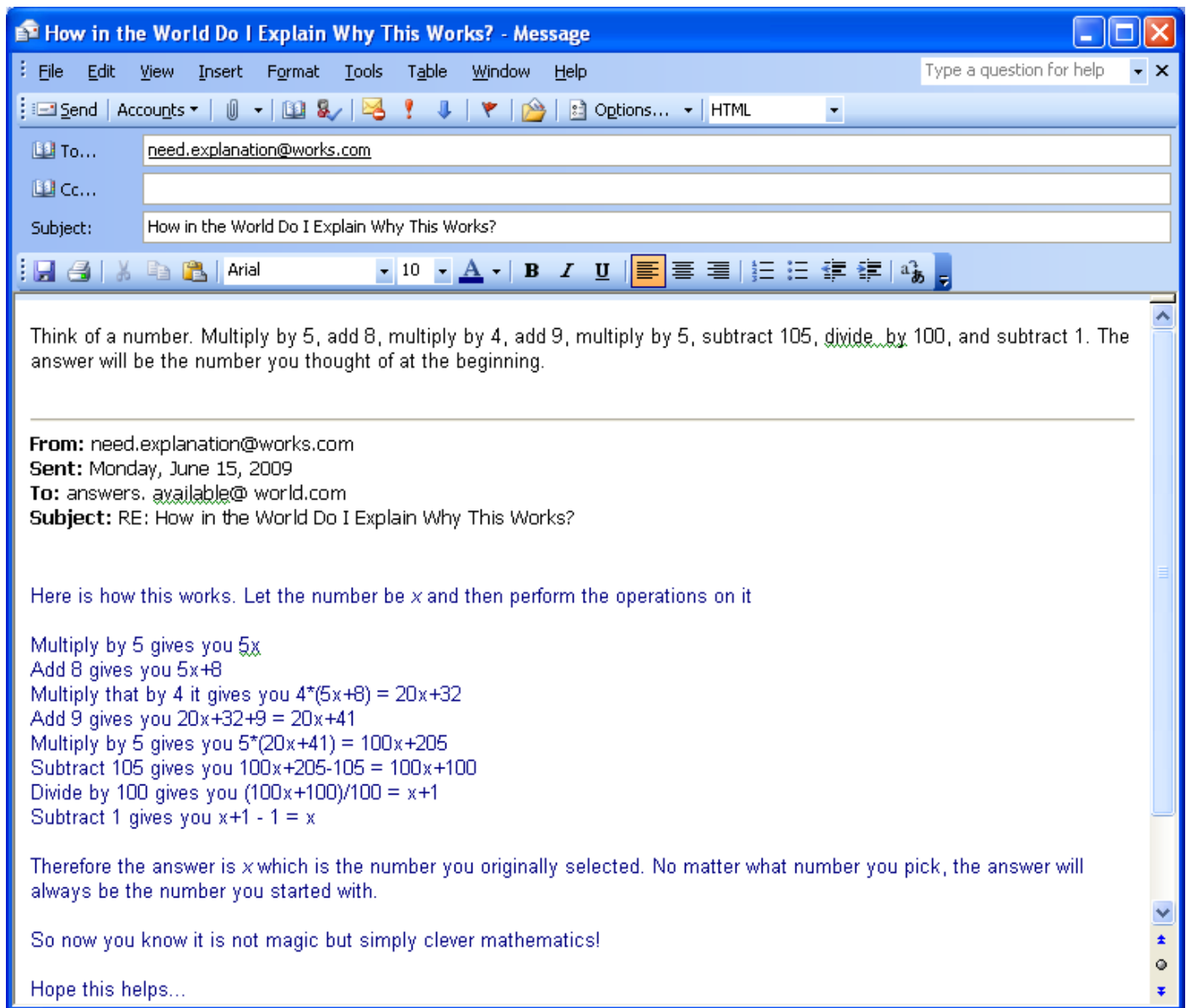
long table. How many people can be seated at this table. How many people could sit at a long table constructed of 10 card tables? How many could sit at a table made with “n” card tables?

A Visit to the County Fair

How much will it cost to ride the amusement rides at the county fair? It costs \$5 to enter the fairgrounds. Amusement rides are \$1.25 per ride. What is the total cost if a visitor rides 1 ride? 5 rides? 10 rides? What cost changes in the problem? What cost stays the same in the problem? If you want to spend no more than \$20, how many rides can you ride? Are there other costs that influence your cost for a day at the fair?

Blanton, M. L. & Kaput, J. J. (October 2003). Developing Elementary Teachers' “Algebra Eyes and Ears”. *Teaching Children Mathematics*, (10)2, 70-77.

Think of a Number Email Handout



The screenshot shows an email client window titled "How in the World Do I Explain Why This Works? - Message". The window has a menu bar (File, Edit, View, Insert, Format, Tools, Table, Window, Help) and a toolbar with various icons. The email header shows the recipient as "need.explanation@works.com", the subject as "How in the World Do I Explain Why This Works?", and the sender as "need.explanation@works.com". The main body of the email contains a math trick and its explanation.

Think of a number. Multiply by 5, add 8, multiply by 4, add 9, multiply by 5, subtract 105, divide by 100, and subtract 1. The answer will be the number you thought of at the beginning.

From: need.explanation@works.com
Sent: Monday, June 15, 2009
To: answers.ayailable@world.com
Subject: RE: How in the World Do I Explain Why This Works?

Here is how this works. Let the number be x and then perform the operations on it

Multiply by 5 gives you $5x$
Add 8 gives you $5x+8$
Multiply that by 4 it gives you $4*(5x+8) = 20x+32$
Add 9 gives you $20x+32+9 = 20x+41$
Multiply by 5 gives you $5*(20x+41) = 100x+205$
Subtract 105 gives you $100x+205-105 = 100x+100$
Divide by 100 gives you $(100x+100)/100 = x+1$
Subtract 1 gives you $x+1 - 1 = x$

Therefore the answer is x which is the number you originally selected. No matter what number you pick, the answer will always be the number you started with.

So now you know it is not magic but simply clever mathematics!

Hope this helps...

Think of a Number * 2

Think of a number.

Add 4.

Multiply by 3.

Subtract 9.

Multiply by 2.

Divide by 6.

Subtract the original number.

What answer did you get?

Think of a Number * 3

Think of a number.

Add 1.

Multiply by 9.

Add the original number.

Subtract 4.

Delete the ones digit.

What number do you obtain?

Think of a Number * 4

Think of a number.

Add 7 to it.

Subtract 2.

Subtract your original number.

Multiply by 4.

Subtract 2.

What is your number?

Think of a Number * 5

Think of a number between 1 and 25.

Double the number.

Add 10.

Divide by 2.

Subtract the original number.

What is the number?

Calendar Algebra

Use a calendar.

Select a 3×3 array of number from a calendar (any month, any year).

Add up the nine numbers (dates) in the array.

Divide the sum by the central number.

What number do you obtain?

YOUR AGE BY CHOCOLATE MATH



**Don't tell me your age; you'd probably lie anyway
-- but the Hershey Man will know!**



**DON'T CHEAT BY SCROLLING
DOWN FIRST!**

**It takes less than a minute. Work
this out as you read.**

**Be sure you don't read the next page until you've worked it
out! This is not one of those waste of time things, it's fun.**



- 1. First of all, pick the number of times a week that you would like to have chocolate (more than once but less than 10).**
- 2. Multiply this number by 2 (just to be bold).**
- 3. Add 5.**
- 4. Multiply it by 50 -- I'll wait while you get the calculator.**
- 5. If you have already had your birthday this year add 1761.
If you haven't, add 1760.**
- 6. Now subtract the four digit year that you were born.**



You should have a three digit number.

The first digit of this was your original number
(i.e., how many times you want to have chocolate each week).



The next two numbers are

YOUR AGE! (Oh YES, it is!!!!)



THIS IS THE ONLY YEAR (2011) IT WILL EVER WORK, SO SPREAD IT AROUND WHILE IT LASTS.





<http://www.wisconline.org>

Simplifying Algebraic Expressions: Addition & Subtraction

Author: Douglas Jensen & Allen Reed

School: Northeast Wisconsin Technical College

Description: Learners read definitions of the terminology associated with algebraic operations and then follow steps to simplify algebraic expressions.

http://www.wisc-online.com/objects/index_tj.asp?objID=GEM1804

Simplifying Algebraic Expressions: Division

Author: Douglas Jensen & Allen Reed

School: Northeast Wisconsin Technical College

Description: Learners read definitions of the terminology associated with algebraic operations and then follow steps to use the fundamental laws of division to simplify algebraic expressions.

http://www.wisc-online.com/objects/index_tj.asp?objID=GEM2104

Simplifying Algebraic Expressions: Multiplication

Author: Douglas Jensen & Allen Reed

School: Northeast Wisconsin Technical College

Description: Learners read definitions of the terminology associated with algebraic operations and then follow steps to use the fundamental laws of multiplication to simplify algebraic expressions.

http://www.wisc-online.com/objects/index_tj.asp?objID=GEM1904

An Introductory Algebraic Word Problem

Author: Kevin Ritzman

School: Fox Valley Technical College

Description: In this animated object, learners follow the steps for solving word problems using algebra. An introductory example is explained.

http://www.wisc-online.com/objects/index_tj.asp?objID=TMH5206