| Hey, Pythagoras! Help Me Understa | THEOREN | Student/Class Goal <br> Students may have heard of the Pythagorean Theorem, but are uncertain how to solve these kinds of problems or why they would be important in their everyday lives. |
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| Outcome (lesson objective) <br> Students will demonstrate their knowledge of the Pythagorean two problems based on real-life situations. | em by creating | Time Frame Up to 5 classes |
| Standard Use Math to Solve Problems and Communicate |  | NRS EFL 3-6 |
| COPs <br> Understand, interpret, and work with pictures, numbers, and symbolic information. | Activity Addresses Components of Performance Students will solve problems using the Pythagorean Theorem and will use substitution to solve formulas. |  |
| Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension. | Students use calculators to solve problems with Pythagorean Theorem. Students will use problem-solving strategies to locate right triangles. |  |
| Define and select data to be used in solving the problem. | Students select the appropriate numbers to substitute in the formula to solve the problem. |  |
| Determine the degree of precision required by the situation. | Students use estimation to determine the place value to round to when using a calculator to find a square root. |  |
| Solve problem using appropriate quantitative procedures and verify that the results are reasonable. | Students estimate to determine if they have completed calculations correctly. Is the missing side an appropriate length? Students use calculators to check results. |  |
| Communicate results using a variety of mathematical representations, including graphs, charts, tables, and algebraic models. | Students communicate results orally and demonstrate their understanding by designing problems with representations, etc. |  |
| Materials <br> Graph paper (white), graph paper (colored) <br> Square tiles or 1" paper squares <br> Rulers <br> Colored paper <br> Calculators <br> Pythagoras Vocabulary Self-Inventory Chart <br> Parts of a Right Triangle Handout <br> Pythagorean Triples Handout <br> Pythagoras Learning Objects |  |  |
| Learner Prior Knowledge <br> Previous experience with square numbers (can be represented by dots in a square array) and square roots (a number when multiplied by itself equals a given number) and the representation of each. Students recognize a right triangle (triangle with a $90^{\circ}$ angle). The lesson Quilting Geometry provides students with a background of identifying angles. |  |  |
| Instructional Activities <br> Step 1 - Each student is given the Pythagoras Vocabulary Self-In identify "I know the word $\dot{-}$, heard of it $\mathcal{O}$ or have no idea $\theta_{,}$," defined before the lesson. Be sure to review what squaring a num <br> Teacher Note Briefly discuss the Greek mathematician, philosop 475 BC , was the first to prove a theorem about right triangles th these web sites for some background information on Pythagoras: <br> Step 2 - Identify the parts of a right triangle. The handout Parts can label. Review the mathematical meaning of the Pythagorea | tory chart with a lis discuss words alr ber means ( $5^{2}=5$ ti and religious leade was known to the B Pythagorean Theor <br> a Right Triangle has heorem. | f words discussed in this lesson. Students known and words that need to be 5). <br> ythagoras. Pythagoras, who died about lonians 1000 years earlier. Check out or Biography of Pythagoras. <br> illustration of a right triangle that students |

In any right triangle, the area of the square whose side is the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares whose sides are the two legs (the two sides that meet at a right angle).

Ask the students to translate the math into plain English. It can be summarized as follows:

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

Step 3 - "Prove" the Pythagorean Theorem by completing the following activity:

1. With a sheet of colored paper, construct a right triangle with the following measurements. Use a corner of the paper for the right angle. One leg will be 3 inches (use a ruler to measure this length) and the other leg will be 4 inches. Remember the legs are the sides of the triangle that form the sides of the right angle. Join the end points of the two legs to form the hypotenuse. Check to be sure it is 5 inches long.
2. Carefully cut out the triangle and glue it in the center of a sheet of paper.
3. Square the length of each side of the triangle ( $3^{2}=9,4^{2}=16$, and $5^{2}=25$ ). Use 9 square tiles to form a square next to the side which is 3 inches long. Use 16 tiles to construct a square next to the side that is 4 inches long. Finally use 25 tiles to construct a square next to the side which is 5 inches long.
4. Using the Pythagorean Theorem, we can see that $16+9=25$. Using our model we can physically see that, too.

Step 4 - Discuss with the students that a triangle with sides of 3,4 , and 5 is a right triangle and called a Pythagorean triple. This next activity will help the students find more Pythagorean triples.

1. Working with a partner, have the students construct (with colored graph paper - centimeter grid paper works well for this) as many perfect squares with areas from 1 to 225 as possible. This may seem like a huge task, so remind the students what a perfect square (an integer that is the square of an integer) is. Have each group cut out and label the perfect squares they have constructed. (i.e. $2^{2}=4$, on a square of $4 ; 3^{2}=9$, on a square of 9 ; etc.)
2. Using the perfect squares, the students will construct right triangles. Review the $(3,4,5)$ triangle they used in the previous activity. Lay the squares out on grid paper of the same size and a different color. Match up the side of the square to the side of the right triangle.
3. Experiment with the perfect squares to discover as many Pythagorean triples / right triangles as possible. Tape the perfect squares next to the sides of the right triangle.
4. Construct a chart to record data about the right triangles using the handout Pythagorean Triples.

Step 5 - Provide practice using the Pythagorean Theorem in real world problems. There are 2 problems included at the end of the lesson that provide good practice. In addition, investigate Pythagorean Theorem problems on the GED test. Have pairs of students solve each problem. Discuss as a class the clues that "told" them the problem required using the Pythagorean Theorem.

Brainstorm with the students situations where right triangles occur:

- Amount of wire needed to run from the top of a pole to a point 6 feet from the base of the pole
- Straight line distance between locations on roads that are perpendicular to one another
- Diagonal distance of a rectangular picture frame or a TV - how screens are measured
- Length of a ramp when you know the height and linear distance it covers
- Diagonal distance across a park
- Area of a lot
- An octagon shaped deck
- A ladder against the side of a house
- Length of the ramp for a moving truck

Be sure to discuss clues that might indicate the problem requires using the Pythagorean Theorem [right angle (square corner, perpendicular to the ground), finding a length of a side of a "triangle" (it might be an imaginary triangle), and diagonals].
Students will create two illustrated problems with solutions that require the Pythagorean Theorem. Below each problem, students will detail a solution that shows in words and numbers how the answer was attained. After assessing each student's problem for situational accuracy, use the problems to create a worksheet for the students to complete.

Teacher Note Since the Pythagorean Theorem is one of the most missed questions on the GED test, it is important that students get lots of practice with problems using the theorem. The next activity gives students an opportunity to explore ways the Pythagorean

Theorem might be presented on the GED.

Step 6 - Students can choose to practice Pythagorean Theorem problems in a GED textbook or by completing the Pythagorean Theory Webquest. Additional practice can be found at Pythagorean Theorem.

Assessment/Evidence (based on outcome)
Pythagorean Triples Handout
Two real-life problems submitted to teacher for quiz development
Completion of student-generated quiz

## Teacher Reflection/Lesson Evaluation

Not yet completed.

## Next Steps

Look at other proofs of the Pythagorean Theorem. Pythagoras Learning Objects will give students additional practice with the theorem, right angles and using a calculator to find solutions.

## Technology Integration

Pythagorean Theorem http://mathforum.org/isaac/problems/pythagthm.html
Biography of Pythagoras http://www-groups.dcs.st-and.ac.uk/~history/Printonly/Pythagoras.html
Pythagorean Theory Webquest http://www.arcytech.org/iava/pythagoras/objective.html
Pythagorean Theorem http://www.mathsisfun.com/pythagoras.html
Using Pythagorean Theorem http://www.pbs.org/wgbh/nova/proof/puzzle/use.html

## Purposeful/Transparent

Students work with the Pythagorean Theorem and real life uses of it throughout the entire lesson.

## Contextual

The lesson emphasizes ways the Pythagorean Theorem is used in real life. Students internalize these ideas when they create their own problems.

## Building Expertise

Students expand their knowledge of powers, roots, and right triangles as they complete the lesson. In addition the lesson increases problem solving skills as students work to find right triangles/Pythagorean Triples.

## Pythagoras Vocabulary Self-Inventory

()

I know the word
$\qquad$ square numbers
$\qquad$ square roots
$\qquad$ right triangle
$\qquad$ right angle
$\qquad$ Pythagorean Theorem
$\qquad$ legs
$\qquad$ hypotenuse
;
Heard of it
$\qquad$ area
$\qquad$ vertex
$\qquad$ Pythagorean Triple
___ solving a right triangle
$\qquad$ isosceles triangle
___ similar triangle
-
©
Have no idea


## Parts of a Right Triangle

Label the right triangle shown below. A word list is included to help you. Some words will be used more than once.

$$
\begin{aligned}
& \text { leg } \\
& \text { hypotenuse } \\
& \text { right angle } \\
& \text { side a } \\
& \text { side } b \\
& \text { side c } \\
& \text { vertex } \\
& a^{2}+b^{2}=c^{2}
\end{aligned}
$$



Pythagorean Theorem

## Pythagorean Triples

Record the data for the right triangles you discovered. Look for patterns in the data.


| $\begin{gathered} \text { Length of } \\ \text { Leg } a \\ \text { (units) } \\ a \\ a \\ \hline \end{gathered}$ | $\begin{gathered} \text { Length of } \\ \text { Leg } b \\ \text { (units) } \\ b \\ \hline \end{gathered}$ | Length of <br> Hypotenuse $\boldsymbol{c}$ <br> (units)$\boldsymbol{c}$ | Area of square with Length $a$ (square units) $a^{2}$ | Area of square with Length $b$ (square units) $b^{2}$ | Area of square with Length $c$ (square units) <br> $c^{2}$ | $a^{2}+b^{2}$ | Column 6\&7 Equal? |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 4 | 5 | 9 | 16 | 25 | 25 | yes |
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## The Pythagorean Theorem and Ladders

You're locked out of your house and the only open window is on the second floor, 25 feet above the ground. You need to borrow a ladder from one of your neighbors. There's a bush along the edge of the house, so you'll have to place the ladder 10 feet from the house. What length of ladder do you need to reach the window?

Give me a hint. Think of the ladder as the hypotenuse of a right triangle. Since you know the length of the other two sides of the triangle, use the Pythagorean theorem to find the length of the hypotenuse: $(10)^{2}+(25)^{2}=c^{2}$

I'm ready to check my answer. You need a ladder that's 27 feet long.

## The Pythagorean Theorem and Baseball

You've just picked up a ground ball at first base, and you see the other team's player running towards third base. How far do you have to throw the ball to get it from first base to third base, and throw the runner out?


Give me a hint. A baseball diamond is actually a square, with right angles at each base. Draw a line from first base to third base. You've created a right triangle with that line as the hypotenuse. Since you know the length of the other two sides of the triangle, use the Pythagorean theorem to find the length of the hypotenuse: $(90)^{2}+(90)^{2}=c^{2}$

I'm ready to check my answer. You need to throw the ball 127.3 feet to get it from first base to third base.
http://www.wisconline.org

## Pythagorean Theorem

Author: Michele Williams
School: Fox Valley Technical College
Description: The student uses the Pythagorean Theorem to find the lengths of the legs and hypotenuse of a right triangle.
http://www.wisc-online.com/objects/index_tj.asp?objID=ABM1501

Right Triangles That Are Special: Using the Pythagorean Theorem
Author: Michele Williams
School: Fox Valley Technical College
Description: Students apply the Pythagorean Theorem to isosceles and equilateral triangles. It is recommended that students complete the Pythagorean Theorem Learning Object before completing this one.
http://www.wisc-online.com/objects/index_tj.asp?objlD=ABM4002

## Pythagorean Theorem Calculations Using a Casio fx-260

Author: Steve Whitmoyer
School: Gateway Technical College
Description: In this animated and interactive object, the learner uses a Casio fx-260 calculator to solve for the magnitude of the resultant and of one of the component forces in a right triangle.
http://www.wisc-online.com/objects/index_tj.asp?objID=ENG9503

