Estimating Percents, Proportions and Square Roots

Student/Class Goal
Students will discover the value of estimation in everyday life and for solving math problems in school.

Outcome (*lesson objective)*
Students mentally estimate percentages, proportions, and the square root of any number.

Time Frame
4 hours

Standard *Use Math to Solve Problems and Communicate*

NRS EFL 6

Number Sense	Benchmarks	Geometry & Measurement	Benchmarks	Processes	Benchmarks
Words to numbers connection		Geometric figures		Word problems	
Calculation	6.1	Coordinate system		Problem solving strategies	
Order of operations	6.2	Perimeter/area/volume formulas		Solutions analysis	
Compare/order numbers		Graphing two-dimensional figures		Calculator	6.29
Estimation	6.4	Measurement relationships		Math terminology/symbols	
Exponents/radical expressions		Pythagorean theorem		Logical progression	
Algebra & Patterns	**Benchmarks**	Measurement applications		Contextual situations	
Patterns/sequences		Measurement conversions		Mathematical material	6.33
Equations/expressions		Rounding		Logical terms	
Linear/nonlinear representations		**Data Analysis & Probability**	**Benchmarks**	Accuracy/precision	
Graphing		Data interpretation		Real-life applications	
Linear equations		Data displays construction		Independence/range/fluency	
Quadratic equations		Central tendency			
		Probabilities			
		Contextual probability			

Materials
white board/chalk board
store sales advertisements
Estimate Proportions Handout
Jars of Sweets Handout
teacher supplied handouts/keys
pencil and paper, calculators

Learner Prior Knowledge
Basic knowledge of percentages, ratios, proportions and square roots.

Instructional Activities
Teacher Note In all activities, first demonstrate to students the examples, then have the students help you solve them. Finally, have students solve examples on their own or in pairs. Handouts can be made prior to class from the examples listed below, websites or classroom resources.

Step 1 - Discuss some different strategies for estimating percents.

1. Some common fractional equivalents to remember.
   100% = 1 (100% of any number equals that number.)

   50% = 1/2 = 0.5 (50% of any number equals half of that number.)

   25% = 1/4 = 0.25 (25% of any number equals one-fourth of that number.)

   10% = 1/10 = 0.1 (10% of any number equals one-tenth of that number.)

1% = 1/100 = 0.01 (1% of any number equals one-hundredth of that number.)

33 1/3% = 1/3 = 0.333… (33 1/3% of any number equals one-third of that number.)

66 2/3 % = 2/3 = 0.666… (66 2/3 % of any number equals two-thirds of that number.)

2. Use a fraction that is close to the percent.

   Example 23% is close to 25% or ¼ of a number.

3. Use either the 10% or 1% of a number method. Round the result, if necessary and then multiply to find the percentage.

   Example 40% of 279. 10% of 279 is 27.9 or about 28. 28 X 4 =112. If a student can't multiply that mentally, then round to 30 X 4 = 120 and use a little less.

4. Use the meaning of percent to estimate.

   Example Estimate 60% of 436. 60% means 60 for every 100 or 6 for every 10. 436 has 4 hundreds and about 3 ½ tens. (60 X 4) + (6 X 3½) = 240 + 21 = 261

5. Estimate 13% of 92. 13% is roughly half of 25%. 25% is half of 50%. Dividing 92 in half is 46, (50% of 92), half of 46 is 23 (25% of 92), half of 23 is 11½ (approximately 13%). The actual answer is 11.96

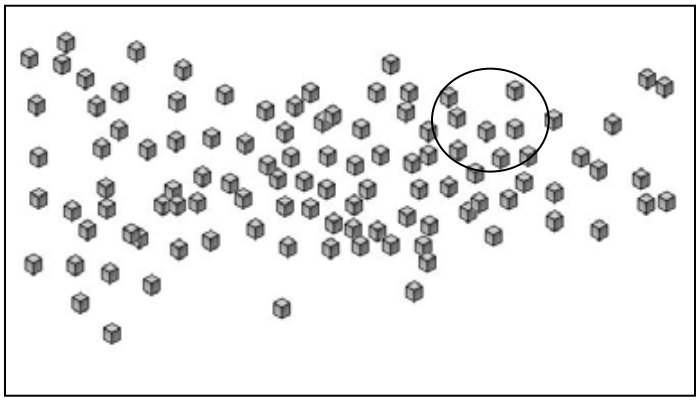
Step 2 - Show students two real life techniques for mentally estimating percents.

Example How to mentally figure a 15% tip at a restaurant. Demonstrate that 15% is equal to 10% + 5% and that 5% is one half of 10%. 10% of any number is that number with the decimal moved one place to the left. A restaurant bill of $28.99 can be rounded up to $30. 10% of 30 is 3 and half of 3 is 1.5. The tip would be $3.00 + $1.50 = $4.50.

Example A jacket is regularly priced at $79.99. It is on sale for 40% off. What is the sale price? Explain that the regular price of anything is equal to 100%. Since there is a reduction of 40%, the buyer pays 60% (100% - 40% = 60%). 60% of $80 ($79.99 rounded up) is 6 X 8 or $48 (or use the 10% method: 10% of 80 = 8 and multiply that by 6).

On each of 10 cards, put a price ($10.99, $79.95, etc.) and a discount in percentages (15%, 30%, and so on). You may want to cut out ads from the newspaper to add a touch of reality. Students turn the cards over, one by one, and write the estimated final price on the back of the card. Check each. Estimate with a calculator. The player whose estimation came closest to the actual price on each item earns one point. To make this more challenging, look for ads that say "Save 20% - now 14.99." Students figure the original price of the item.

Step 3 - Demonstrate to students one way to estimate proportions.



Pass out *Estimate Proportions* handout of image above or project it and ask students to guess how many cubes there are. Draw a circle around approximately ten cubes that look to be of average space. Estimate how many circles it would take to cover all the cubes and set up the proportion. There are 108 cubes in the diagram.

Capture-recapture is a statistical method used to estimate the size of a population. Fish and wildlife management experts, demographers, and scientists use this and other techniques to find the number of people or animals in a region. Students can complete How Many Fish in the Pond?

Teacher Note Additional real life applications can be found at Math Focal Points.

Pass out *Jars of Sweets* handout to students and go over each question with them. Try to estimate what proportion of the jar contains sweets.

Jar 1 = 1/2, Jar 2 = 1/8, Jar 3 = 3/4, Jar 4 = 1/5, Jar 5 = 1/4

If the jar held 100 sweets estimate:

Jar 1 = 50, Jar 2 = 12, Jar 3 = 75, Jar 4 = 20, Jar 5 = 25

If the jar held 200 sweets estimate:

Jar 1 = 100, Jar 2 = 25, Jar 3 = 150, Jar 4 = 40, Jar 5 = 50

Use these sample five questions and have students work on their own or in pairs, then go over the answers with them.

1. About how much is 62½ % of 42?

A. 15 B. 30

C. 25 D. 5

Answer: C Use the 10% method – 10% of 42 is 4.2. 4.2 (or 4) x 6 is about 25

2. Estimate 67% of 28.

A. 15 B. 25

C. 20 D. 10

Answer: C Use the 10% method – 10% of 28 is 2.8 (or 3) x 6 is about 18, or knowing 67% is about 2/3, 28/3 is about 9 so 1/3 is about 9 and 2/3 is about 18

3. Estimate 12½ % of 72.

A. 63 B. 45

C. 27 D. 9

Answer: D If students know that 12.5% is 1/8, divide 72 by 8 or use the 10% method

4. Estimate 33% of 119.

A. 70 B. 40

C. 50 D. 60

Answer: B Students should know 33% is about 1/3. Round 119 to 120 and divide by 3

5. Your friend Janet has 1 1/5 cups of rice. About how much rice do you have if you have 30% the amount of rice your friend Janet has?

A. 4 cups B. 9/25 cup

C. 1 1/25 cups D. 2 1/5 cups

Answer: B The answer has to be less than 1 1/5 cups because you have about 1/3 of what Janet has. That eliminates A & D. It also makes C highly unlikely because it is close to 1 1/5. Or, estimate 1/3 of a cup is 9/27 + the 1/5 brings you close to 9/25.

Step 4 - Show students one way to estimate square roots.

Example Have the students estimate the square root of 415. First determine the pair of perfect squares the number falls between. The square root of 415 falls between the square root of 400 which is 20 and the square root of 441 which is 21. So the square root of 415 has to be between 20 and 21. To get a better estimate, there are 41 numbers between 400 and 441. 415 is 15 numbers past 400, so use 15/41 which equals 30/82 or approximately 3/8 which equals 0.375. So the square root of 415 is about 20.375. By using this method, students do not need to memorize any algorithms and it usually provides accuracy to the nearest tenth or hundredth.

Assessment/Evidence (*based on outcome)*
Completion of problems and real-life application activity.

Teacher Reflection/Lesson Evaluation
This lesson has not yet been field tested.

Next Steps
Additional practice problems of estimating percents, proportions and square roots can be online or in classroom resources.

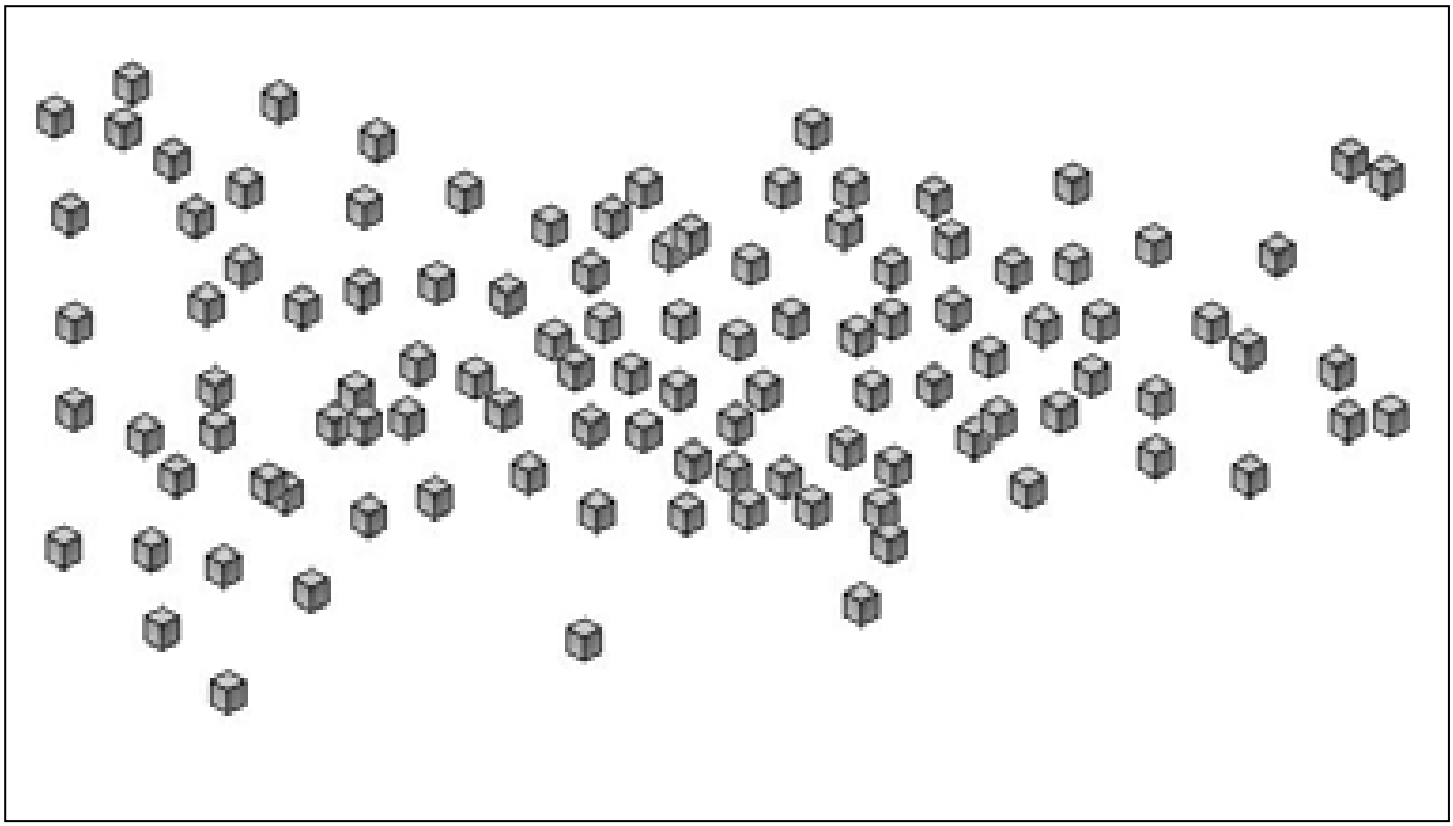
Technology Integration
How Many Fish in the Pond? http://www.figurethis.org/challenges/c52/challenge.htm
Math Focal Points http://msteacher.org/epubs/math/math15/ratio.aspx

Purposeful/Transparent
Students will discover the usefulness of estimation when eating out or shopping.

Contextual
Students practice estimating a tip, the amount off a sale item, and the proportion of an amount so they can mentally determine accuracy when calculated.

Building Expertise
The teacher builds on prior knowledge of estimation and allows students to practice concepts throughout the lesson until students are comfortable and ready to move on to next concept.

Estimate Proportions



Jars of Sweets

Try to estimate what proportion of the jar contains sweets.

Jar 1 Jar 2 Jar 3 Jar 4 Jar 5

If a full jar holds 100 sweets, how many sweets do you estimate there are in each jar? How did you make your estimate?

Jar 1 Jar 2 Jar 3 Jar 4 Jar 5

If a full jar holds 200 sweets, how many sweets do you estimate there are in each jar? How did you make your estimate?

Jar 1 Jar 2 Jar 3 Jar 4 Jar 5

NAME _____ DATE _____

9-8 Percent and Estimation (Pages 462–466)

When an exact answer is not needed, you can estimate percentages.

Estimating Percents	Method 1: With the fraction method, use a fraction that is close to the percent. For example, 24% is about 25% or $\frac{1}{4}$. Method 2: With the 1% method, find 1% of the number. Round the result, if necessary, and then multiply to find the percentage. Method 3: Use the meaning of percent to estimate.

EXAMPLES

A Estimate 40% of 183 using the 1% method.

1% of 183 is 1.83 or about 2.
So 40% of 183 is about 40 × 2 or 80.

B Estimate 60% of 537 using the meaning of percent.

60% means 60 for every 100 or 6 for every 10. 537 has 5 hundreds and about 4 tens (37 ≈ 40).
(60 × 5) + (6 × 4) = 300 + 24 or 324.

Try These Together

1. What fraction could you use to estimate 34% of a number?
   HINT: $\frac{1}{3}$ is about 33%.

2. Estimate a percent for 29 out of 40.
   HINT: 29 out of 40 is close to 30 out of 40.

PRACTICE

Write the fraction, mixed number, or whole number you could use to estimate.

3. 110%
4. 22%
5. 41%
6. 8.5%
7. 49%
8. 430%

Estimate.

9. 13% of 79
10. 58% of 190
11. 98% of 11
12. 41% of 20
13. 109% of 500
14. 73% of 21
15. 87% of 90
16. 31% of 87

Estimate each percent.

17. 19 out of 39
18. 20 out of 55
19. 4 out of 300

20. **Nutrition** If a package of 4 cookies has 205 Calories and 30% of the Calories come from fat, estimate how many of the 205 Calories are from fat.

21. **Standardized Test Practice** Choose the best estimate for 11% of 833.

A 0.083 **B** 0.83 **C** 8.3 **D** 83

Answers: Estimates may vary. 1. $\frac{1}{3}$ 2. 75% 3. 1 4. $\frac{1}{5}$ 5. $\frac{2}{5}$ 6. $\frac{1}{10}$ 7. $\frac{1}{2}$ 8. 4 9. 8 10. 120 11. 11 12. 8 13. 550 14. 15 15. 81 16. 27 17. 50% 18. 40% or 33% 19. 1% 20. 60 calories 21. C