DOUBLE, DOUBLE: LOOKING AT THE EFFECT OF CHANGE ON PERIMETER, AREA AND VOLUME

Student/Class Goal
Students want to become more familiar and comfortable with math formulas before taking the GED test.

Outcome (*lesson objective*)
Students will demonstrate how changes in the dimensions of squares, rectangles, and circles affect the perimeter/circumference, area and volume of shapes.

Time Frame
3 hours over 2-3 classes

Standard *Use Math to Solve Problems and Communicate*

NRS EFL 3-6 depending on shapes

COPs	**Activity Addresses Components of Performance**
Understand, interpret, and work with pictures, numbers, and symbolic information.	Students use numbers to calculate perimeter, area and volume of shapes using formulas. Students construct tables with data.
Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension.	Students will perform mathematical operations and use calculators.
Define and select data to be used in solving the problem.	Students will decide which values should be substituted into the formula
Determine the degree of precision required by the situation.	Students will estimate the change in the area of the figures. Students will round numbers when working with circle problems.
Solve problem using appropriate quantitative procedures and verify that the results are reasonable.	Students will use drawings, models and mathematical formulas to confirm results.
Communicate results using a variety of mathematical representations, including graphs, charts, tables, and algebraic models.	Results will be communicated in words, illustrations, and mathematical formulas.

Materials
Square/Triangular Graph paper
Manipulatives (square tiles, pattern blocks)
Rulers and yardsticks
Paper and scissors
Calculator
Double, Double Perimeter Handout
Double, Double Volume Handout

Learner Prior Knowledge
Students need a basic understanding the formulas for perimeter/circumference, area, and volume. They must also be able to use the formulas to calculate an answer. These formulas are found on the GED formula sheet available in GED math text books or at GED Testing Service.

Instructional Activities
Step 1 - "Double, Double" with Area
Introduce a situation to the class where area or square shapes is involved. Quilts blocks and patios with pavers are good examples as they are constructed in square units. The following story is a possible example, but use whatever situation your students can relate to.

Susie Quiltmaker is making a quilt for her living room sofa. The quilt will be square with a 5 by 5 block layout. Before she sews the blocks together, she lays the quilt blocks on the design wall. When she looks at the layout, Susie decides that she would like a larger quilt. The 5 by 5 quilt is too small for her family to snuggle up under on a cold night. She decides to make the quilt twice as big. So instead of making her quilt 5 blocks by 5 blocks, she will double both dimensions to make it 10 blocks by 10 blocks.

Discuss Susie's quilt with the class. Did Susie double the size of her quilt? Distribute square tiles to pairs of students. Ask each group to construct both the original 5x5 quilt and the larger quilt using square tiles to represent blocks. Next ask them to discuss what they see. Did Susie double the size of her quilt? (Yes, she more than doubled it.) What is the approximate relationship between the 2 quilts? (4 times larger) How might you prove this relationship? (Count the square tiles). Note: This activity could also be done with graph paper rather than square tiles.

Summarize the discussion by noting that when we doubled the length of the sides of a square, the area of the new square was four times the original area. Ask students if they think the same thing will happen with other regular shapes. Ask them to look at equilateral triangles. (Use the link at the end of the lesson to create triangle graph paper in the paper and triangle size you would like. The green equilateral triangles in a set of pattern blocks work well, too.) Ask students to construct an equilateral triangle with a side of 3 and then another with a side of 6. Ask what happens to the area of the triangle. Did the relationship hold true? Note: Don't try to use the formula for the area of a triangle for this one. Just use one small triangle as having an area of 1 unit.

Finally, ask students to consider circles. Ask students what happens to the area of a circle if we double the diameter or radius. Ask students to consider buying pizzas -- Which is the better buy, two 6-inch pizzas or one 12-inch pizza? Using the formula for the area of circles, ask students to calculate the area of a 6-inch pizza and a 12-inch pizza. What effect does doubling the radius have on the area of a circle? Which is the better buy? Note that only one dimension was changed in this shape. The radius is squared (r x r) so we are multiplying two dimensions.

Step 2 - Ask students to use graph paper or square tiles to experiment with the area of rectangles:

- Construct a small rectangle (2 by 4). Now double the measure of the length and width (4 by 8) to construct another rectangle.
- Look at the size of the new rectangle and compare it to the original. Approximately how much larger is the new rectangle?
- Find the area of both shapes. What is the actual difference? Does our "double, double" relationship work with rectangles? Encourage students to sketch several different sized rectangles so the class is confident the relationship always works.
- Now investigate what would happen if we double just one dimension. Construct a rectangle that is 2 by 8 or 4 by 4 (doubling just the length or width). What happens to the rectangle now?

Next ask students to look at triangles. Ask them to use regular graph paper to construct a triangle with a base of 5 and a height of 4. Then ask them to construct a new triangle doubling the base and the height (base of 10 and height of 8). Ask them to look at the two shapes and compare their areas visually and then to find the area of each triangle mathematically. What is their relationship? Then ask them to investigate what happens if only one dimension of the triangle is changed (base or height). What happens? Finally, discuss with the students what would happen with parallelograms and trapezoids. Construct these shapes on graph paper and see.

Step 3 – Double, Double" with Perimeter
List the results of the previous activities. What happened to the area shapes when two dimensions were doubled? Now ask students what they think will happen to the perimeter of a shape when two dimensions are doubled.

Ask students to think about Susie Quiltmaker, who now needs to bind her quilts. Her original quilt was going to be 5 feet by 5 feet. How much binding would she need for the quilt? Ask students to sketch or use a formula to calculate the length of the binding she needs. Her doubled quilt was 10 feet by 10 feet. How much binding does she need for this quilt? Record the results on a chart from the *Double, Double Perimeter* handout. What happens to the perimeter when each dimension is doubled?

Ask students what will happen if only one dimension is doubled. Ask them to try to see what happens. Complete the handout, but only double one dimension. Is there a constant relationship between the perimeters?

Step 4 **-** "Double, Double" with Volume
Examine the effect doubling two dimensions of an object has on the volume of the enlarged shape.
Complete *Double, Double Volume* handout to explore how rectangular containers, square pyramids, cylinders and cones are affected.

Step 5 **–** To summarize, ask students to work in small groups to explain how doubling affects the perimeter, area and volume of shapes. Ask each group to select how they will explain what happens (writing, pictures, math formulas or a combination of the three) and how they will present the information (group presentation, poster, or essay). Provide time for students to complete their activity and invite sharing at the next class session.

Assessment/Evidence (*based on outcome)*
Group presentations

Teacher Reflection/Lesson Evaluation
Not yet completed.

Next Steps

Technology Integration
GED Formula Sheet http://www.acenet.edu/Content/NavigationMenu/ged/test/math_formulas.htm
Triangle Graph Paper PDF Generator http://incompetech.com/graphpaper/triangle/

Math Literacy News: Measurement http://literacy.kent.edu/Oasis/Pubs/mathwinter03.pdf

Purposeful/Transparent
Lesson provides lots of practice using formulas for perimeter, area and volume. Students use various models and diagrams to investigate the formulas.

Contextual
Student ideas are encouraged as they apply this lesson to real-life situations such as quilting, tiles, etc.

Building Expertise
Students reinforce a basic knowledge of geometric formulas and expand that knowledge to look at basic number principles.

DOUBLE, DOUBLE PERIMETER

Experiment with squares and rectangles to see what happens to the perimeter of a shape when its dimensions (length and width) are doubled.

	Original Shape				Doubled Shape				
Shape	**Length**	**Width**	**Perimeter 1**		**Length**	**Width**	**Perimeter 2**		**P2÷P1**
Square	3	3	12		6	6	24		24/12=2

On the lines below, describe your observations about what happens when the length and width of a shape are doubled.

DOUBLE, DOUBLE VOLUME

Explore what happens to the volume of rectangular containers, square pyramids, cylinders, and cones when two of the dimensions in the shape are doubled.

	Original Shape					Doubled Shape				
Shape	**Length**	**Width**	**Height**	**Volume**		**Length**	**Width**	**Height**	**Volume**	**V2÷V1**
Rectangular Container	2	2	2	8		4	4	2	32	32÷8=4

	Original Shape				Doubled Shape			
Shape	**Radius**	**Height**	**Volume**		**Radius**	**Height**	**Volume**	**V2 ÷V1**
Cylinder	2	4	50.24		4	4	200.96	200.96 ÷ 50.24 = 4

On the lines below, describe your observations about what happens when the two dimensions of a shape are doubled.

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Formulas

AREA of a:

square	Area = side^2
rectangle	Area = length × width
parallelogram	Area = base × height
triangle	Area = $^1/_2$ × base × height
trapezoid	Area = $^1/_2$ × (base$_1$ + base$_2$) × height
circle	Area = π × radius^2; π is approximately equal to 3.14.

PERIMETER of a:

square	Perimeter = 4 × side
rectangle	Perimeter = 2 × length + 2 × width
triangle	Perimeter = side$_1$ + side$_2$ + side$_3$
CIRCUMFERENCE of a circle	Circumference = π × diameter; π is approximately equal to 3.14.

VOLUME of a:

cube	Volume = edge^3
rectangular solid	Volume = length × width × height
square pyramid	Volume = $^1/_3$ × (base edge)^2 × height
cylinder	Volume = π × radius^2 × height; π is approximately equal to 3.1□.
Cone	Volume = $^1/_3$ × π × radius^2 × height; π is approximately equal to 3.14.

COORDINATE GEOMETRY

distance between points = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
(x_1, y_1) and (x_2, y_2) are two points in a plane.

slope of a line = $\dfrac{y_2 - y_1}{x_2 - x_1}$
(x_1, y_1) and (x_2, y_2) are two points on the line.

PYTHAGOREAN RELATIONSHIP

$a^2 + b^2 = c^2$; *a* and *b* are legs and *c* the hypotenuse of a right triangle.

MEASURES OF CENTRAL TENDENCY

mean = $\dfrac{x_1 + x_2 + \ldots + x_n}{n}$**,** where the *x*'s are the values for which a mean is desired, and *n* is the total number of values for *x*.

median = the middle value of an odd number of *ordered* scores, and halfway between the two middle values of an even number of *ordered* scores.

SIMPLE INTEREST	interest = principle × rate × time
DISTANCE	distance = rate × time
TOTAL COST	total cost = (number of units) × (price per unit)

December 2003—Issue #12

MATH Literacy News

MEASUREMENT

Nancy Markus

Measurement is a topic that often gives students problems and yet it is one of the most relevant and useful aspects of our curriculum. What makes measurement so difficult and what can we do to help our students master the skills and concepts necessary to become proficient in this area?

First we must understand what measurement is. John A. Van de Walle in "Elementary and Middle School Mathematics: Teaching Developmentally" states that "Measurement is a number that indicates a comparison between the attribute of the object being measured and the same attribute of a given unit of measure." Richard Cherry and Garland John Gates at the 6th Annual State-Wide Math Workshop, decided that measurement is "the application of a mutually agreed upon system (metric, standard, or arbitrary) used to quantify a trait (such as length, area, or volume)." Whatever definition we use, measurement remains a problem area in many adult education classrooms.

John Van de Walle gives us these BIG IDEAS needed to develop measurement concepts:

1. Measurement involves a comparison with a unit that has the same attribute as the item which is being measured (length, volume, weight, etc.) There are many ways to make these comparisons. To measure any thing meaningfully, the attribute to be measured must be understood.
2. Measurement instruments are devices that replace the need for actually measuring units in making comparisons.
3. Area and volume formulas are ways of using length measures to count more easily the area or volume units in an object without actually using area or volume units.

DISCUSSION OF THE TOPIC

Discussion can introduce new topics and help students build on prior knowledge. It is encouraged that the teacher discuss each topic to introduce it to the class. What is known about measurement? What problems do the students have? Students must realize that in order to measure something, they must:

- decide on the attribute to be measured
- select a unit that has that attribute
- compare the units by filling, covering, matching , or some other method with the attribute of the object being measured

Often students are not quite sure what the difference is between linear, square, and cubic measurements. Linear units are one-dimensional and are used for perimeter and circumference. Examples of linear units are inches, feet, yards, meters, or kilometers. Area units are two-dimensional and cover a flat space. Examples of area units are square inches, square feet or square meters. Volume or capacity is measured in three-dimensional units and describe how much can fill a space. Cubic units might be cubic inches, cubic feet or cubic yards. Identifying and writing the correct units is an important part of measuring correctly. It is important to have square inches, cubic inches, etc. for the students to see and hold.

MEASURING LENGTH

Make a ruler

Students are given a strip of paper, and asked to measure it using square inch tiles. How could we measure without the tiles? By marking our paper with the inch marks, of course! In a first

Ohio Literacy Resource Center
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Math Literacy News is a publication of the Ohio Literacy Resource Center
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ruler, the students can count the units. Numbers on the rulers will help students count the inches; numbers are often placed in the center of the units. When the numbers are placed at the *end* of each unit, the ruler becomes more standard and is actually a number line!

After students have made their own rulers, it is important to measure many things with them. A natural progress will be to add half inches and perhaps quarter inches as needed.

Make an enlarged inch
Using a large sheet of paper (8 ½ by 11 inch is a good choice) students can construct an enlarged inch. Fold the paper in half and indicate with a long line that the middle point is ½. By folding in half again, the student can mark off 1/4, 2/4, and 3/4 . (The 2/4 will be written under the ½.) Continue with eighths and sixteenths. Compare the enlarged inch with a standard ruler. Check to see how halves, fourths, eighths, and sixteenths are differentiated on the standard ruler. Are some of the lines longer than others? How many lines are in between each inch mark? Why are there only 15 lines *between* each inch?

Measure items with the enlarged inch. Desks, books, boxes, etc. are all good choices. Students should become accustomed to measuring with this enlarged inch before trying to measure with the standard ruler.

Establish benchmarks
Have students find an inch on their bodies (between two knuckles?) A centimeter (a fingernail?). Find a yard (often from the tip of the nose to the end of the hand). Find a meter. How wide is a hand? (Note: the standard hand measured sideways for horses is 4 inches including the thumb. How close is each student's hand to this standard measure?) How long is each student's foot? (Make sure to take shoes off!) An extension to this activity is to compare length of feet to shoe sizes. How is shoe size determined? Make a graph with the class results. Bring in measurements of family members and/or other students in the building to increase the database.

Estimate measurements
How big is the room? Can the students estimate the length, width, and height of the room? What different ways can be used to estimate these measurements? Are there standard room heights? Standard door heights? Students should be encouraged to *use benchmarks, chunk, use subdivisions, and iterate a unit mentally or physically.* It may be easier to use *chunks* such as windows, cabinets, etc. Or if a wall is completely blank, *subdividing* it mentally in half, then in fourths and even eighths may help the students reach a manageable unit to estimate. A student might *iterate* single units such as bricks or even another student's height. By knowing how long one's foot is, it is possible to pace off one-foot lengths.

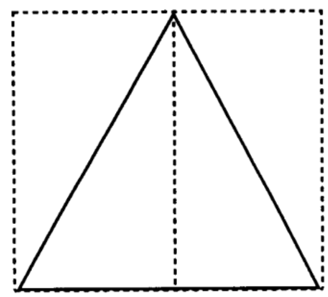
It is important to discuss the different ways used to estimate. Students can measure to check their estimates but this should not be the focus.

MEASURING AREA

Build a model
Using square tiles, build different sized rectangles to explore. Students can compare perimeter and area; although the same dimensions are used, perimeter and area are not the same. Students can explore the idea that the closer one gets to a square figure, the smaller the perimeter will be for a given area.

Build a square foot with the square inch tiles and then cut one out of cardboard. Since a square *anything* is a square that is *anything* on each side, a square foot, must be one square foot (or twelve inches) on each side. Students can see that it would take 144 square inches to fill in each one square foot. Nine square feet can be put together to model a square yard.

Develop formulas
Using graph and/or grid paper, cut out a rectangle and compare the formula on the formula sheet to the dimensions of the figure as well as the square unit. Do the same for a square (and be sure to discuss the difference in the formulas). Finally take a rectangle and or square and cut it in half. Decide why the formula for a triangle is ½ bh. Make a triangle such as below:



Can the student "see" the rectangles that each side can form? Remember that relying on formulas with no understanding of where those formulas come from is the road to misunderstanding!

MEASURING VOLUME

Build a model
Using a grid or graph paper, build a one-layer structure using cubes. Note that the volume would be length times width times one since there is one layer. Now add a layer. Note that the new volume is length times width times two since there are two layers. Continue to build different rectangular solid structures using the cubes.

Determine volumes
Using a cubic inch, students can estimate and measure volumes of a variety of boxes and/or items in the classroom such as a briefcase or file cabinet.

Establish benchmarks
Have each student hold the cubic inch to feel its size. A cubic inch block is a very useful instrument to help students understand volume. A cubic foot can be built using six of the square feet that were cut from cardboard. Students can see that it would take 1728 cubic inches to equal one cubic foot! Note: A cubic foot is approximately the size of a gallon of milk. A cubic yard can be compared to the size of an average washing machine.

MEASURING TIME

Have students stand with their eyes closed. Have them sit down when one minute has passed. Keep track of the times that each student sits down. Discuss the one thousand one, one thousand two, method.

MEASURING ANGLES

Use a paper plate circle graph
Put two paper plates that are exactly the same except for the colors together, by cutting one radius on each and fitting them together. The plates can be rotated to show angles, fractions, decimals, and percents.

Make a protractor
Cut a basket coffee filter in half. Fold the filter in half and mark the fold with a magic marker and the measure of 90 degrees. Fold in half again to indicate 45 degrees. The filter may have creases. Students can figure out how many degrees each crease indicates. These can be drawn on the filter also. Compare your coffee filter protractor with a standard protractor. Practice measuring various angles with the coffee filter protractor. Paper can be cut out or angles can be drawn on paper to be measured. The "protractor" can be set over the angle to be measured since it is translucent.

Use benchmarks
The 90 degree angle is the easiest to use as a benchmark. Every piece of standard paper has four 90 degree angles that can be fitted into any angle to be measured.

BIBLIOGRAPHY

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